천정크레인의 로드위치 제어를 위한 슬라이딩 모드 컨트롤
Sliding Mode Position Control of Overhead Crane

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Abstract We propose a sliding mode load position controller for an overhead crane to cope with large load variations. The controller is developed using the input-output decoupling control laws obtained by the dynamic extension technique. Due to unknown load mass, it is difficult to design a decoupling control law which exactly cancels out all the nonlinearities. To overcome the problem, we derive a sliding mode controller. The simulation results show that the proposed controller performs well under significant load variations.

Keywords Overhead crane, Load position control, sliding mode control, Input-output linearization, Dynamic extension

1. Introduction

The overhead cranes are widely used in many industrial fields such as factories, warehouses, shipyards, harbors and so on. They are indispensable in transporting heavy load and basically consist of just two simple mechanisms: trolley and hoister. But it is not easy to accurately control them, because they are highly nonlinear, under-actuated and come with uncertainties. The significant nonlinearity comes mostly from varying rope lengths. If the trolley of the overhead crane moves without hoisting and lowering, then its model becomes virtually linear. With an additional assumption that the rope angle from the perpendicular is close to zero, it can be transformed into a linear system which is relatively easy to control. But a skillful human operator moves trolley with hoisting or lowering for better performance, and hence using the nonlinear crane model in controller design is useful to achieve high control performance. Usually the overhead crane comes with two control inputs: trolley position and rope length, but the degree of freedom is three: trolley position, rope length and rope angle from the perpendicular. Therefore, it is an underactuated system, which causes complexity in controller design. And the unknown load mass adds an inevitable significant uncertainty to the overhead crane and a robust controller is required to cope with it.

Since the late 1970s, overhead crane control has been one of the very popular research topics, so there have been reported a number of research results. The crane control techniques can be categorized into two classes: anti-swing control and load position control. Since the objective of crane operation is to transport the load from one place to another quickly and smoothly, undesirable swing motion must be controlled until the load reaches the destination. For this reason, many anti-swing control techniques have been developed [1-11]. Actually, anti-swing control can be considered as an alternative and indirect way of load position control, because it concentrates on eliminating undesired sway while the trolley and the hoister follow the desired trajectories. If there is no undesired sway, then the load position can be controlled by using one of the trolley and hoister control techniques [6,12].

On the other hand, the direct load position control schemes are less developed [13,14]. Among the output regulation or tracking control techniques, those based on the input-output linearization are useful for a class of nonlinear systems. But it is difficult to apply an input-output linearization technique to the overhead crane system, because it comes with ill-defined relative degree [13] However, Cheng [14] overcame the difficulty using the dynamic extension method and obtained a mapping between the load position and the control input, which enables us to control the load position directly. But the input-output linearization method is sensitive to system uncertainties, because they jeopardize exact nonlinearity cancelation and the uncontrolled nonlinearity can even cause instability [16]. For crane systems, the unknown load weight is a main source of uncertainty and the use of robust control technique is important. To this aim, a time delay control (TDC) technique was suggested in [14] however, it requires extra conditions.

We propose in this paper a robust load position controller for an overhead crane system. It is designed to take the unknown load mass as a system uncertainty and to endure load mass variations. We derive a conventional sliding mode control (SMC) technique which is one of the popular robust control methods, in that it is very simple to implement and robust under system uncertainties. The simulation tests are used to validate the performance of the controller.

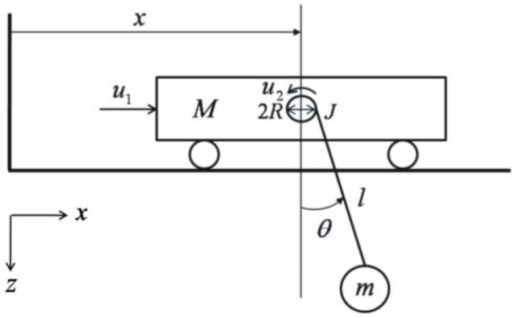

Fig. 1. Overhead crane

An overhead crane system can be represented as nonlinear state-space equations [15]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \quad (1)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}) \quad (2)$$

where

$$\mathbf{x} = \begin{bmatrix} x & l & \theta & \dot{x} & \dot{l} & \dot{\theta} \end{bmatrix}^T \quad (3)$$

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \dot{x} \\ \dot{l} \\ \dot{\theta} \\ \frac{mJ(l\dot{\theta}^2 + g\cos\theta)\sin\theta}{Q} \\ \frac{mMR^2(l\dot{\theta}^2 + g\cos\theta)}{Q} \\ \frac{-mJ(l\dot{\theta}^2 + g\cos\theta)\sin\theta\cos\theta - \left(2\dot{l}\dot{\theta} + g\sin\theta\right)Q}{Ql} \end{bmatrix} \quad (4)$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{J+mR^2}{Q} & \frac{mR\sin\theta}{Q} \\ \frac{-mR^2\sin\theta}{Q} & \frac{-(MR+mR\sin^2\theta)}{Q} \\ \frac{-(J+mR^2)\cos\theta}{Ql} & \frac{-mR\sin\theta\cos\theta}{Ql} \end{bmatrix} \quad (5)$$

$$\mathbf{h}(\mathbf{x}) = \begin{bmatrix} x + l\sin\theta & l\cos\theta \end{bmatrix}^T \quad (6)$$

$$Q = M(J + mR^2) + mJ\sin^2\theta \quad (7)$$

Here x is the trolley position; l is the rope length; θ is the rope angle from the perpendicular; $\dot{x}$, $\dot{l}$ and $\dot{\theta}$ are the time derivatives of x, l and θ respectively; m is the load mass; M is the trolley mass; R is the radius of the winch; J is the moment of inertia of the winch; g is the gravity acceleration.

By differentiating **y** twice, we obtain

$$\ddot{\mathbf{y}} = \begin{bmatrix} -\frac{MJ(l\dot{\theta}^2 + g\cos\theta)\sin\theta}{Q} \\ g - \frac{MJ(l\dot{\theta}^2 + g\cos\theta)\cos\theta}{Q} \end{bmatrix} + \begin{bmatrix} \frac{J\sin^2\theta}{Q} & \frac{-MR\sin\theta}{Q} \\ \frac{J\sin\theta\cos\theta}{Q} & \frac{-MR\cos\theta}{Q} \end{bmatrix}\mathbf{u} \quad (8)$$

But the pre-multiplied matrix of control input u in (8) is singular and we cannot obtain a straightforward decoupling control law. In order to obtain a nonsingular decoupling matrix, we choose a new control input $q = (q_1, q_2)^T$ as follows:

$$\mathbf{u} = \begin{bmatrix} q_1 \\ \frac{Q(g-z_1)}{MR\cos\theta} - \frac{J(l\dot{\theta}^2 + g\cos\theta)}{R} + \frac{J\sin\theta}{MR} q_1 \end{bmatrix} \quad (9)$$

and introduce z_1, z_2 as extended states:

$$\dot{z}_1 = z_2; \quad \dot{z}_2 = q_2. \quad (10)$$

Then substituting (9) into (8) and simplify the equation, we obtain

$$\ddot{\mathbf{y}} = \begin{bmatrix} -g\tan\theta \\ 0 \end{bmatrix} + \begin{bmatrix} \tan\theta \\ 1 \end{bmatrix} z_1 \quad (11)$$

Note from (11) that z_1 becomes the vertical acceleration of load [14, 19].

Introducing the extended state $\mathbf{x}_e$, we obtain

$$\dot{\mathbf{x}}_e = \mathbf{f}_e(\mathbf{x}_e) + m\mathbf{\Delta}_0(\mathbf{x}_e) + \mathbf{G}_e(\mathbf{x}_e)\mathbf{q} \quad (12)$$

$$\mathbf{y} = \mathbf{h}_e(\mathbf{x}_e) \quad (13)$$

where

$$\mathbf{x}_e = \begin{bmatrix} x \\ l \\ \theta \\ \dot{x} \\ \dot{l} \\ \dot{\theta} \\ z_1 \\ z_2 \end{bmatrix}, \ \mathbf{f}(\mathbf{x}_e) = \begin{bmatrix} \dot{x} \\ \dot{l} \\ \dot{\theta} \\ 0 \\ \frac{l\dot{\theta}^2\cos\theta + z_1 - g\sin^2\theta}{\cos\theta} \\ \frac{-(2\dot{l}\dot{\theta} + g\sin\theta)}{l} \\ z_2 \\ 0 \end{bmatrix} \quad (14)$$

$$\Delta_0(\mathbf{x}_e) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{(g-z_1)\tan\theta}{M} \\ \frac{-(g-z_1)\sin\theta\tan\theta}{M} \\ \frac{-(g-z_1)\sin\theta}{Ml} \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

$$\mathbf{G}_e(\mathbf{x}_e) = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{M} & \frac{-\sin\theta}{M} & \frac{-\cos\theta}{Ml} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \quad (16)$$

$$\mathbf{h}(\mathbf{x}_e) = \begin{bmatrix} x + l\sin\theta & l\cos\theta \end{bmatrix}^T. \quad (17)$$

By differentiating **y** in (13) four times, we obtain the following input-output mapping

$$\mathbf{y}^{(4)}=\begin{bmatrix}\frac{\left(-2l\dot{\theta}^2\tan\theta+2\dot{l}\dot{\theta}+g\sin\theta\right)(g-z_1)+2l\dot{\theta}z_2}{l\cos^2\theta}\\ 0\end{bmatrix}+\begin{bmatrix}\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}\\ 0\end{bmatrix}+\begin{bmatrix}\frac{g-z_1}{Ml\cos\theta} & \tan\theta\\ 0 & 1\end{bmatrix}\mathbf{q}, \quad (18)$$

where it is reasonable to assume that $|z_1| \ll g$, $0 \le l_{\min} \le l \le l_{\max}$ and $|\theta| < \pi/2$, and where l_{min} and l_{max} is an allowable minimum and maximum rope length. Note here that the relative degree and the order of the extended dynamic system are the same in this case which is eight. Internal dynamics then does not exist in this case [18] and the error dynamics becomes

$$\dot{\mathbf{e}}_1=\mathbf{e}_2; \quad \dot{\mathbf{e}}_2=\mathbf{e}_3; \quad \dot{\mathbf{e}}_3=\mathbf{e}_4;$$

$$\dot{\mathbf{e}}_4=\mathbf{y}^{(4)}-\mathbf{y}_d^{(4)}=\begin{bmatrix}\frac{\left(-2l\dot{\theta}^2\tan\theta+2\dot{l}\dot{\theta}+g\sin\theta\right)(g-z_1)+2l\dot{\theta}z_2}{l\cos^2\theta}\\ 0\end{bmatrix}+\begin{bmatrix}\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}\\ 0\end{bmatrix}+\begin{bmatrix}\frac{g-z_1}{Ml\cos\theta} & \tan\theta\\ 0 & 1\end{bmatrix}\mathbf{q}-\mathbf{y}_d^{(4)}, \quad (19)$$

where

$$\mathbf{y}_d=[y_{d1} \quad y_{d2}]^T;\ \mathbf{e}_1=[e_{11} \quad e_{12}]^T=\mathbf{y}-\mathbf{y}_d;$$
$$\mathbf{e}_2=[e_{21} \quad e_{22}]^T=\dot{\mathbf{y}}-\dot{\mathbf{y}}_d;\ \mathbf{e}_3=[e_{31} \quad e_{32}]^T=\ddot{\mathbf{y}}-\ddot{\mathbf{y}}_d;$$
$$\mathbf{e}_4=[e_{41} \quad e_{42}]^T=\dddot{\mathbf{y}}-\dddot{\mathbf{y}}_d.$$

2. Controller Design

In controller design, we assume that all the state variables ($x, l, \theta, \dot{x}, \dot{l}, \dot{\theta}$) and the vertical load acceleration and jerk ($\ddot{y}_{d2}, \dddot{y}_{d2}$) are available; desired trajectory y_d is sufficiently smooth; load mass is the only unknown system parameter.

$$q_2=y_{d2}^{(4)}-k_{42}e_{42}-k_{32}e_{32}-k_{22}e_{22}-k_{12}e_{12} \quad (20)$$

Note from (10) that we can determine the extended state variable z_1 and z_2 by integrating q_2 in (20) and they will be used to determine q_1.

In designing a sliding surface, we choose k_{31}, k_{21} and k_{11} such that the polynomial

$$s^3+k_{31}s^2+k_{21}s+k_{11} \quad (21)$$

is Hurwitz. Then the sliding surface becomes

$$s=k_{11}e_{11}+k_{21}e_{21}+k_{31}e_{31}+e_{41}=0 \quad (22)$$

Along with its time derivative

$$\dot{s}=k_{11}e_{21}+k_{21}e_{31}+k_{31}e_{41}+\dot{e}_{41}=k_{11}e_{21}+k_{21}e_{31}+k_{31}e_{41}+\frac{\left(-2l\dot{\theta}^2\tan\theta+2\dot{l}\dot{\theta}+g\sin\theta\right)(g-z_1)+2l\dot{\theta}z_2}{l\cos^2\theta}+\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}+\frac{g-z_1}{Ml\cos\theta}q_1+q_2\tan\theta \quad (23)$$

If we choose a Lyapunov function candidate

$$V(s)=\tfrac{1}{2}s^2, \quad (24)$$

then

$$\dot{V}=s\dot{s}=s\Big(k_{11}e_{21}+k_{21}e_{31}+k_{31}e_{41}+\frac{\left(-2l\dot{\theta}^2\tan\theta+2\dot{l}\dot{\theta}+g\sin\theta\right)(g-z_1)+2l\dot{\theta}z_2}{l\cos^2\theta}+\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}+\frac{g-z_1}{Ml\cos\theta}q_1+q_2\tan\theta\Big) \quad (25)$$

Choosing

$$q_1=-\frac{Ml\cos\theta}{g-z_1}\Big(k_{11}e_{21}+k_{21}e_{31}+k_{31}e_{41}+\frac{\left(-2l\dot{\theta}^2\tan\theta+2\dot{l}\dot{\theta}+g\sin\theta\right)(g-z_1)+2l\dot{\theta}z_2}{l\cos^2\theta}+q_2\tan\theta+v\Big), \quad (26)$$

we have

$$\dot{V}=s\dot{s}=s\left(v+\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}\right). \quad (27)$$

Let

$$v=-\left(\alpha+\left|\frac{m_{\max}(g-z_1)^2\tan\theta}{Ml\cos\theta}\right|\right)\mathrm{sgn}(s) \quad (28)$$

then (26) becomes

$$\dot{V}=s\dot{s}=s\left(\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}+v\right)=s\left[-\left(\alpha+\left|\frac{m_{\max}(g-z_1)^2\tan\theta}{Ml\cos\theta}\right|\right)\mathrm{sgn}(s)+\frac{m(g-z_1)^2\tan\theta}{Ml\cos\theta}\right]<0 \quad (29)$$

And consequently $s \to 0$ as $t \to \infty$.

3. Simulation Studies

We simulated the controllers with the following parameters: M = 1.06kg; J = 0.005kgm^2; R = 0.005m; G = 9.81m/s^2; $\hat{m}$ =5kg; $m_{\max}$ = 5kg, where $\hat{m}$ is the expected load mass of input-output linearization controller (IOLC) and $m_{\max}$ is the maximum allowable load mass of the proposed sliding mode controller (SMC). Both controllers have the same q_2 control law which has four times multiple poles at (10, 0). The IOLC has four times multiple poles at (3, 0) for the control law of q_1 and the sliding surface of the SMC has triple pole at (3, 0).

The IOLC worked well when there is no difference between the load mass and the expected one in Fig. 2. But if there is a mass difference which acts as a control input noise and it may degrade performance. Fig. 3 shows the case. However, compared with the IOLC, the proposed SMC showed robustness to the large load mass variation and stable performance (Fig. 2, 3, 4 and 5). But it showed a little longer settling time and larger overshoot than those of the IOLC. They come from the characteristics of a sliding mode controller which efforts to decrease error by forcing pre-defined s to be zero, i.e., before reaching the sliding surface the error is not guaranteed to decrease. Hence, it takes time a little longer to settle down and shows large overshoot.

And the proposed SMC showed a good anti-swing control performance, even though; Anti-swing control was not the objective of the control. The extended system has no internal dynamics and the proposed SMC is a full state variable feedback controller, which implies all of the state variables are observable and controllable. Thus, actually the state variable θ was under control not to diverge.

The control input of the proposed SMC in Fig. 5 and 7 shows a typical chattering, especially in u_1. Because u_1 is equal to the control input q_1 which was design by sliding mode control technique. Chattering causes large power consumption and heat generation. It can be alleviated by using other type of signum function such as a saturation function.

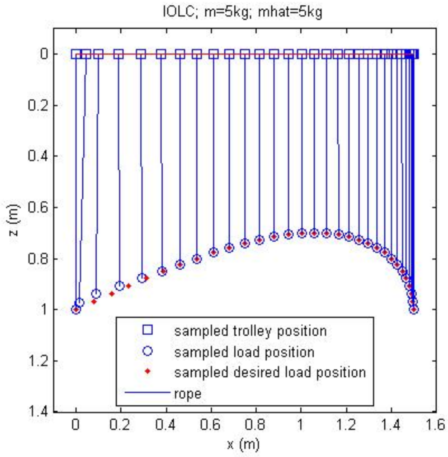

Fig. 2. Load position trajectory: m=5kg, $\hat{m}$ =5kg (IOLC)

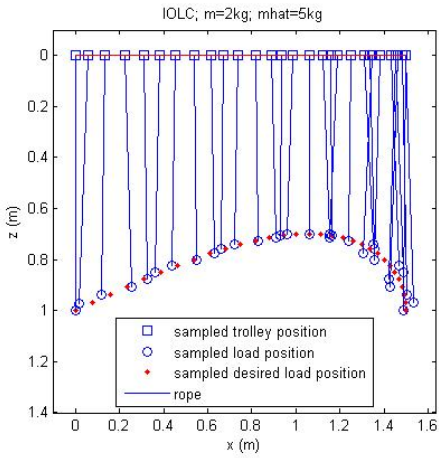

Fig. 3. Load position trajectory: m=2kg, $\hat{m}$ =5kg (IOLC)

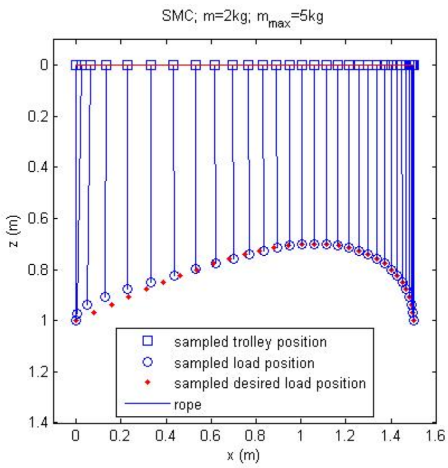

Fig. 4. Load position trajectory: m=2kg, $m_{\max}$=5kg (SMC)

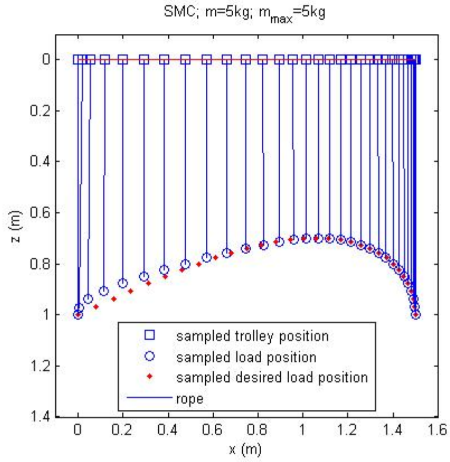

Fig. 4. Load position trajectory: m=5kg, $m_{\max}$=5kg (SMC)

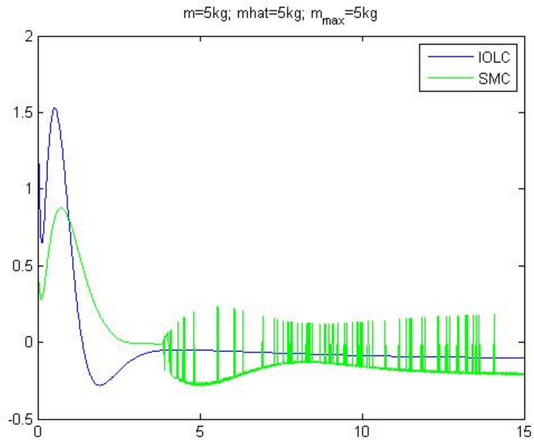

Fig. 5. Control input u_1: m=5kg

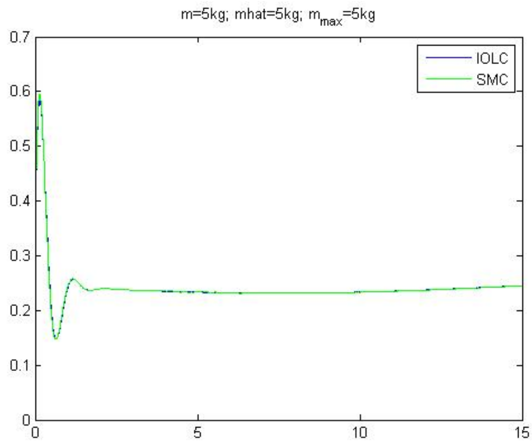

Fig. 6. Control input u_2: m=5kg

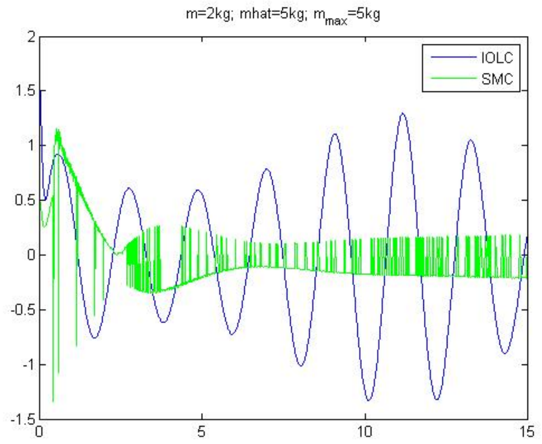

Fig. 7. Control input u_1: m=2kg

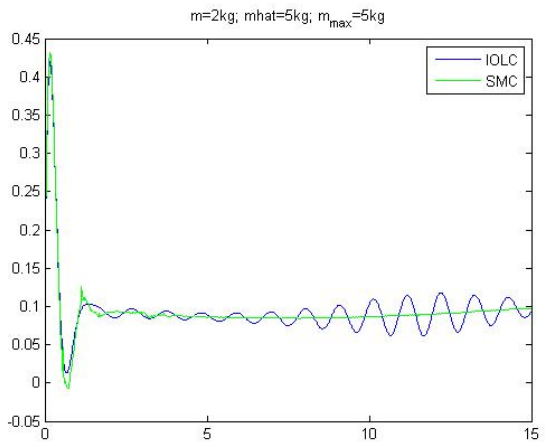

Fig. 8. Control input u_2: m=2kg

4. Conclusions

We proposed a sliding mode controller for load position control of an overhead crane. It was designed to treat unknown load mass as a system uncertainty. We showed it tracks the desired trajectory asymptotically by using a Lyapunov function analysis. In the simulation, it showed robust control performance against load mass variation, however, also showed chattering which is a main drawback of sliding mode controller. It will be considered as future work.

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