Design and Stability Analysis of TSK–type Full–Scale Fuzzy PID Controllers

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***Abstract*—In this paper, we present the design and stability analysis of Takagi–Sugeno–Kang (TSK)–type full–scale fuzzy proportional–integral–derivative (PID) controller. We reveal the analytical structure of a TSK–type full–scale fuzzy PID controller relative to the conventional PID controller. This controller consists of two input fuzzy sets for each of the three input variables, singleton fuzzifier, center average defuzzifier and the MIN operator for the AND operation. We prove that this type of fuzzy controller is actually a nonlinear PID controller with the variable gains. Then, we present BIBO stability analysis of TSK–type full–scale fuzzy PID controller using Small–Gain Theorem.**

I. INTRODUCTION

Conventional PID controllers are the most common industrial controllers due to their simple structure and their robust performance. Because these PID controllers are linear controllers, they become less effective if the plant under control is nonlinear or comes with time–delay [1], [2].

To overcome this problem, many researchers have suggested PID controllers using the fuzzy structure to increase the accuracy and convergence speed of the controllers, and to reduce their overshoot. Because fuzzy controllers can implement human experience in their structure, they have been applied to several plants for which mathematical models are difficult to derive but which human experts understand intuitively.

Many fuzzy PID controllers have been proposed and analyzed. But most of them have been limited to proportional–integral (PI) [3], [4], [5] and proportional–derivative (PD) [1], [4], [6] types. Some are of the PID type, but most of these are in fact designed with decomposed forms such as PD+I [7], PI+D [8], PI+PD [2], [9], [10] and P+I+D [11], [12].

Non–decomposed (here, 'full–scale') fuzzy PID controllers are rare due to their analytic complexity. Some full–scale fuzzy PID controllers have been proposed [13], [14], [15]. However, all combinations of the input variables were not considered. If the input variables of the controller are not contained in the defined combinations, stability cannot be guaranteed. The stability analysis of the Mamdani–type full–scale fuzzy controller for all combinations of input variables was studied in [16].

In this paper, we will study the TSK–type full–scale fuzzy PID controllers for all combinations of input variables. The TSK fuzzy controllers was developed in 1985 [17] and has been widely used. Recently, many researchers have explored the analytical relationship between TSK–type fuzzy PI, PD, PID controllers and conventional PID controllers [18], [19], [20], [21]. Also, there are only a few stability results on the TSK–type fuzzy control systems [22], [23], [24].

We first establish the relationship between a TSK–type full–scale fuzzy PID controller and the conventional PID controller. The structure of the TSK–type full–scale fuzzy PID controller is analytically derived. The derived controller is a discrete–time version of the PID controller in incremental form. We employ only two input fuzzy sets for each of the input variables to simplify the design procedure and reduce the complexity as [13], [14], [15]. Then we employ the Small Gain Theorem to analyze the BIBO stability of the TSK–type full–scale fuzzy PID control system.

II. CONFIGURATION OF TSK–TYPE FULL–SCALE FUZZY PID CONTROLLERS

In developing the controller, we use the following notations:

$$\begin{aligned}
e(nT) &= SP - y(nT),\\
\bar{e}(nT) &= K_e \times e(nT),\\
r(nT) &= e(nT) - e((n-1)T),\\
\bar{r}(nT) &= K_r \times r(nT),\\
a(nT) &= r(nT) - r((n-1)T)\\
&= e(nT) - 2e((n-1)T) + e((n-2)T),\\
\bar{a}(nT) &= K_a \times a(nT)
\end{aligned}$$

where n is a nonnegative integer and T is the sampling period. SP is the set point, $y(nT)$ is the plant output, $e(nT)$ is the output error, $r(nT)$ is the output error rate, and $a(nT)$ is the output error acceleration. K_e, K_r and K_a are the input scales of $e(nT)$, $r(nT)$ and $a(nT)$ for normalizing the input variables. $\bar{e}(nT)$ is the normalized error, $\bar{r}(nT)$ is the normalized error rate, and $\bar{a}(nT)$ is the normalized error acceleration. The discrete–time PID controller in its original form (position form) [25], [26] is given as

$$u(nT) = K_P e(nT) + K_I \sum_{i=0}^{n} e(iT) + K_D r(nT)$$

where $u(nT)$ denotes the controller output, K_P is the controller's P–gain, K_I is its I–gain and K_D is its D–gain. The output of the PID controller in incremental form is:

$$\begin{aligned}\Delta u(nT) =& u(nT) - u((n-1)T)\\ =& K_P e(nT) + K_I \sum_{i=0}^{n} e(iT) + K_D r(nT)\\ &- K_P e((n-1)T)\\ &- K_I \sum_{i=0}^{n-1} e(iT) - K_D r((n-1)T)\\ =& K_P r(nT) + K_I e(nT) + K_D a(nT).\end{aligned}$$

The TSK–type full–scale fuzzy PID controller in its incremental form consists of three inputs: the error, the error rate, and the error acceleration. Therefore, if we use the incremental PID controller, the actual control action $u(nT)$ can be obtained as:

$$u(nT) = u((n-1)T) + \Delta u(nT).$$

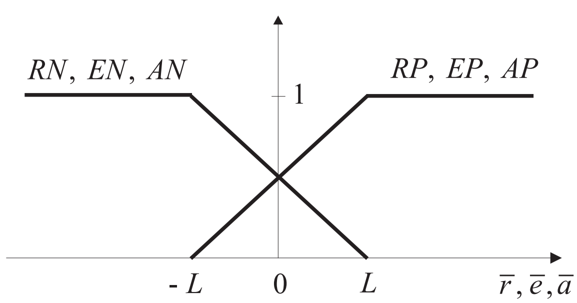


Fig. 1. Input membership functions.

Each of the input variables is described by using two fuzzy sets: Error Negative (EN), Error Positive (EP) for the error; Rate Negative (RN), Rate Positive (RP) for the error rate; Acceleration Negative (AN) and Acceleration Positive (AP) for the error acceleration. These fuzzy sets are characterized by membership functions (Fig. 1). Three membership functions in $[-L, L]$ can be described as:

$$\begin{aligned}\mu_{EN}(\bar{e}) &= \frac{L - \bar{e}(nT)}{2L},\ \mu_{EP}(\bar{e}) = \frac{L + \bar{e}(nT)}{2L};\\ \mu_{RN}(\bar{r}) &= \frac{L - \bar{r}(nT)}{2L},\ \mu_{RP}(\bar{r}) = \frac{L + \bar{r}(nT)}{2L};\\ \mu_{AN}(\bar{a}) &= \frac{L - \bar{a}(nT)}{2L},\ \mu_{AP}(\bar{a}) = \frac{L + \bar{a}(nT)}{2L};\end{aligned}$$

where $\mu_{EN}(\bar{e}) + \mu_{EP}(\bar{e}) = 1$, $\mu_{RN}(\bar{r}) + \mu_{RP}(\bar{r}) = 1$, $\mu_{AN}(\bar{a}) + \mu_{AP}(\bar{a}) = 1$, and outside of $[-L, L]$, the membership value is either 0 or 1.

Assume that the TSK–type full–scale fuzzy PID controller applies eight fuzzy IF–THEN rules:

R1 : IF $\bar{r}(nT)$ is RN AND $\bar{e}(nT)$ is EN AND $\bar{a}(nT)$ is AN THEN $\Delta u_1(nT)$ is $\alpha_1 \bar{r}(nT) + \beta_1 \bar{e}(nT) + \gamma_1 \bar{a}(nT)$

R2 : IF $\bar{r}(nT)$ is RN AND $\bar{e}(nT)$ is EN AND $\bar{a}(nT)$ is AP THEN $\Delta u_2(nT)$ is $\alpha_2 \bar{r}(nT) + \beta_2 \bar{e}(nT) + \gamma_2 \bar{a}(nT)$

R3 : IF $\bar{r}(nT)$ is RP AND $\bar{e}(nT)$ is EN AND $\bar{a}(nT)$ is AN THEN $\Delta u_3(nT)$ is $\alpha_3 \bar{r}(nT) + \beta_3 \bar{e}(nT) + \gamma_3 \bar{a}(nT)$

R4 : IF $\bar{r}(nT)$ is RP AND $\bar{e}(nT)$ is EN AND $\bar{a}(nT)$ is AP THEN $\Delta u_4(nT)$ is $\alpha_4 \bar{r}(nT) + \beta_4 \bar{e}(nT) + \gamma_4 \bar{a}(nT)$

R5 : IF $\bar{r}(nT)$ is RN AND $\bar{e}(nT)$ is EP AND $\bar{a}(nT)$ is AN THEN $\Delta u_5(nT)$ is $\alpha_5 \bar{r}(nT) + \beta_5 \bar{e}(nT) + \gamma_5 \bar{a}(nT)$

R6 : IF $\bar{r}(nT)$ is RN AND $\bar{e}(nT)$ is EP AND $\bar{a}(nT)$ is AP THEN $\Delta u_6(nT)$ is $\alpha_6 \bar{r}(nT) + \beta_6 \bar{e}(nT) + \gamma_6 \bar{a}(nT)$

R7 : IF $\bar{r}(nT)$ is RP AND $\bar{e}(nT)$ is EP AND $\bar{a}(nT)$ is AN THEN $\Delta u_7(nT)$ is $\alpha_7 \bar{r}(nT) + \beta_7 \bar{e}(nT) + \gamma_7 \bar{a}(nT)$

R8 : IF $\bar{r}(nT)$ is RP AND $\bar{e}(nT)$ is EP AND $\bar{a}(nT)$ is AP THEN $\Delta u_8(nT)$ is $\alpha_8 \bar{r}(nT) + \beta_8 \bar{e}(nT) + \gamma_8 \bar{a}(nT)$

where $\Delta u_i(nT)$ for $i = 1, \cdots, 8$ is the contribution of each rule to the change of the control output, and α_i, β_i and γ_i for $i = 1, \cdots, 8$ are 24 design parameters, all of which are nonnegative to avoid positive feedback of fuzzy control systems [27]. The MIN operator is used for the AND operation; the MIN membership functions for the eight rules are:

$$\begin{aligned}\mu_1 &= \text{MIN}(\mu_{RN}, \mu_{EN}, \mu_{AN}) \quad \text{for Rule } 1,\\ \mu_2 &= \text{MIN}(\mu_{RN}, \mu_{EN}, \mu_{AP}) \quad \text{for Rule } 2,\\ \mu_3 &= \text{MIN}(\mu_{RP}, \mu_{EN}, \mu_{AN}) \quad \text{for Rule } 3,\\ \mu_4 &= \text{MIN}(\mu_{RP}, \mu_{EN}, \mu_{AP}) \quad \text{for Rule } 4,\\ \mu_5 &= \text{MIN}(\mu_{RN}, \mu_{EP}, \mu_{AN}) \quad \text{for Rule } 5,\\ \mu_6 &= \text{MIN}(\mu_{RN}, \mu_{EP}, \mu_{AP}) \quad \text{for Rule } 6,\\ \mu_7 &= \text{MIN}(\mu_{RP}, \mu_{EP}, \mu_{AN}) \quad \text{for Rule } 7,\\ \mu_8 &= \text{MIN}(\mu_{RP}, \mu_{EP}, \mu_{AP}) \quad \text{for Rule } 8.\end{aligned} \tag{1}$$

From the IF–THEN rules, we know that the output of the TSK–type full–scale fuzzy PID controllers is the same as the output of the conventional PID controller with three gains K_p, K_i and K_d if the following conditions are satisfied:

$$\begin{aligned}&\alpha_1 = \cdots = \alpha_8 = \alpha,\\ &\beta_1 = \cdots = \beta_8 = \beta,\\ &\gamma_1 = \cdots = \gamma_8 = \gamma,\\ &\alpha \times K_r = K_p,\\ &\beta \times K_e = K_i,\\ &\gamma \times K_a = K_d.\end{aligned}$$

Finally, the controller output $\Delta u(nT)$ is obtained by the center average defuzzifier:

$$\Delta u\,(nT) = \frac{\sum_{i=1}^{8} \Delta u_i\,(nT)\,\mu_i}{\sum_{i=1}^{8} \mu_i}.$$

We derive analytically the structure of the TSK–type full–scale fuzzy PID controller:

$$
\begin{aligned}
&\Delta u(nT)\\
&= \{\textstyle\sum_{i=1}^{8} (\alpha_i \bar{r}(nT) + \beta_i \bar{e}(nT) + \gamma_i \bar{a}(nT))\, \mu_i\} / \textstyle\sum_{i=1}^{8} \mu_i \\
&= \sum_{i=1}^{8} \frac{\mu_i}{\sum_{i=1}^{8} \mu_i} (\alpha_i \bar{r}(nT) + \beta_i \bar{e}(nT) + \gamma_i \bar{a}(nT)) \\
&= \frac{\sum_{i=1}^{8} \mu_i \alpha_i}{\sum_{i=1}^{8} \mu_i} \bar{r}(nT) + \frac{\sum_{i=1}^{8} \mu_i \beta_i}{\sum_{i=1}^{8} \mu_i} \bar{e}(nT) + \frac{\sum_{i=1}^{8} \mu_i \gamma_i}{\sum_{i=1}^{8} \mu_i} \bar{a}(nT) \\
&= K_P(\bar{r},\bar{e},\bar{a}) \bar{r}(nT) + K_I(\bar{r},\bar{e},\bar{a}) \bar{e}(nT) + K_D(\bar{r},\bar{e},\bar{a}) \bar{a}(nT)
\end{aligned} \quad (2)
$$

where

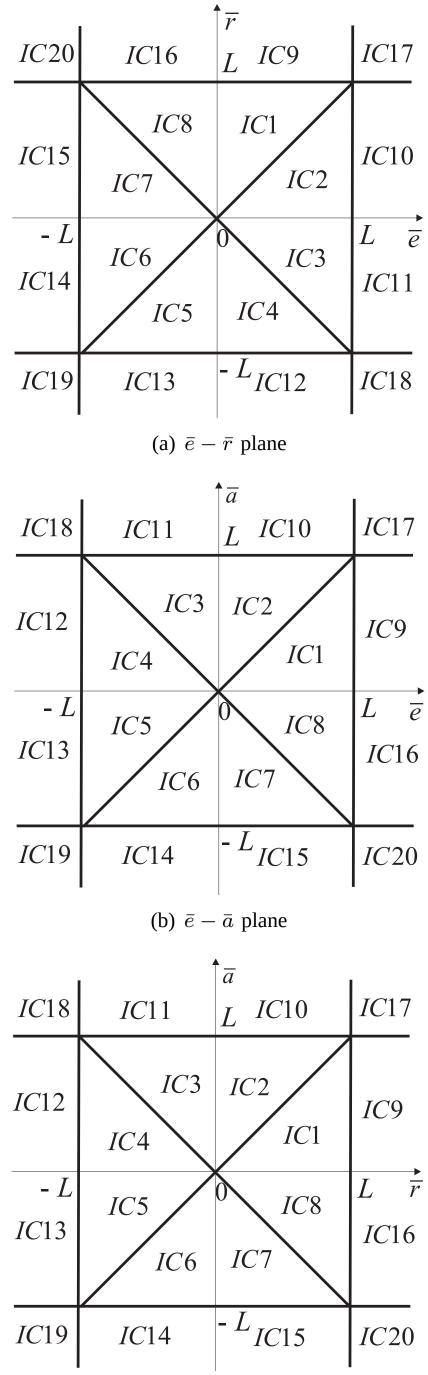
$$
\begin{aligned}
K_P(\bar{r},\bar{e},\bar{a}) &= \textstyle\sum_{i=1}^{8} \mu_i \alpha_i / \sum_{i=1}^{8} \mu_i, \\
K_I(\bar{r},\bar{e},\bar{a}) &= \textstyle\sum_{i=1}^{8} \mu_i \beta_i / \sum_{i=1}^{8} \mu_i, \\
K_D(\bar{r},\bar{e},\bar{a}) &= \textstyle\sum_{i=1}^{8} \mu_i \gamma_i / \sum_{i=1}^{8} \mu_i.
\end{aligned}
$$

$K_P(\bar{r},\bar{e},\bar{a})$, $K_I(\bar{r},\bar{e},\bar{a})$ and $K_D(\bar{r},\bar{e},\bar{a})$ of the TSK–type full–scale fuzzy PID controller are not constants; they vary over time according to the input variables.

III. Input–output Relationship of TSK–type Full–Scale Fuzzy PID Controllers

The detailed input–output relationship of the TSK–type full–scale fuzzy PID controller can be derived by exhaustively considering the combinations of input variables $\bar{r}(nT)$, $\bar{e}(nT)$ and $\bar{a}(nT)$. Due to the use of AND operator, the input space must be divided into a number of regions in such a way that in each region a unique analytical inequality relationship can be obtained for each fuzzy rule between the two membership functions being ANDed. Two methods can express these combinations; the table method [1] and the cube method [13].

The table method has been commonly used for two–input fuzzy controllers, i.e., PI, PD or decomposed fuzzy controllers, because the input combinations (ICs) of two variables can be easily expressed. Because the input membership functions we use in this paper are expressed as in Fig. 1, the number of ICs of two input fuzzy controllers is 20 (Fig. 2). The table method is useful to describe the relationships between input variables, so the input–output relationship of the controller can be easily derived. But in the case of a full–scale fuzzy PID controller, three tables are necessary to express the input combinations: one for the $\bar{e}$–$\bar{r}$ plane, one for the $\bar{e}$–$\bar{a}$ plane, and one for the $\bar{r}$–$\bar{a}$ plane. From each plane, three ICs are chosen, then combined to form a three–dimensional cell (IC in $\bar{e}$–$\bar{r}$, IC in $\bar{e}$–$\bar{a}$, and IC in $\bar{r}$–$\bar{a}$), so the table consists of $20 \times 20 \times 20 = 8000$ cells. However, most of the input combinations are not valid. A cell is valid if the relationships among input variables $\bar{r}, \bar{e}$ and $\bar{a}$ do not conflict. For example, consider Cell(1,2,2) (Fig. 2). Each component in the cell represents IC1 (Fig. 2 (a)), IC2 (Fig. 2 (b)), and IC2 (Fig. 2 (c)). In IC1, $\bar{r} \geq \bar{e}$; in IC2, $\bar{a} \geq \bar{e}$; in IC2, $\bar{a} \geq \bar{r}$. These results yield $\bar{a} \geq \bar{r} \geq \bar{e}$ which does not conflict. But as another example, consider Cell(1,1,3) (Fig. 2). In this case, $\bar{r} \geq \bar{e}$, $\bar{e} \geq \bar{a}$ and $\bar{a} \geq \bar{r}$, which is not valid because a contradiction occurs in the interior of the cell. The table method is useful to describe the relationships among variables, but determining valid cells is time–consuming and tedious.



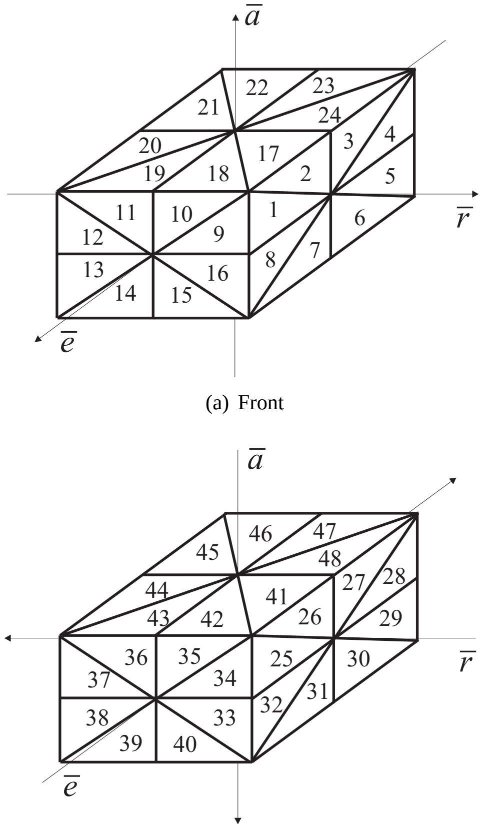
(a) $\bar{e} - \bar{r}$ plane

(b) $\bar{e} - \bar{a}$ plane

(c) $\bar{r} - \bar{a}$ plane

Fig. 2. Input combinations (table method).

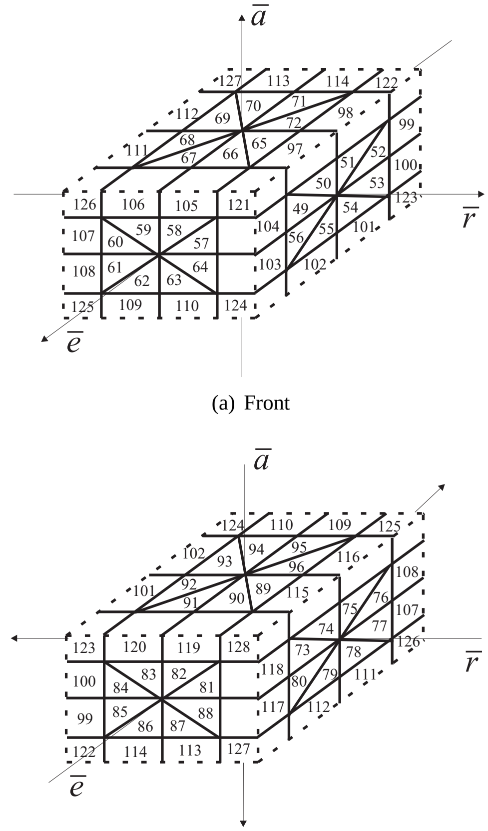
To remedy this disadvantage of the table method, the cube method can be used. Intuitively, the cube method is convenient to express combinations of three variables as a



(a) Front

(b) Back

Fig. 3. Input sectors in $[-L, L] \times [-L, L] \times [-L, L]$ (cube method).



(a) Front

(b) Back

Fig. 4. Input sectors outside of $[-L, L] \times [-L, L] \times [-L, L]$ (cube method).

three–dimensional unit sector (Figs. 3, 4). For example, Sector 1 consists of four points which are the origin and the visible three vertices (Fig. 5). This method reveals 128 sectors: 48 sectors in $[-L, L] \times [-L, L] \times [-L, L]$ and 80 sectors outside of $[-L, L] \times [-L, L] \times [-L, L]$ (Figs. 3, 4). In the cube method, finding valid sectors is unnecessary, because all sectors are valid. However, the cube method is a complicated and confusing way of expressing the relationships among the three variables.

Each of the two methods has its own merit and demerit: the table method is convenient to analyze the relationship of input variables, but the valid cells must be identified; the cube method expresses all valid sectors, but finding relationships among variables is difficult. Therefore, we use both methods to exploit the merits of each.

Now, we match a sector in the cube to the cell in the table (Table I and II). To do this, we divided the variables into four categories:

1) All variables are in $[-L, L] \times [-L, L] \times [-L, L]$ (Sectors from 1 to 48).
2) One variable is outside $[-L, L] \times [-L, L] \times [-L, L]$ and two variables are in $[-L, L] \times [-L, L] \times [-L, L]$ (Sectors from 49 to 96).
3) One variable is in $[-L, L] \times [-L, L] \times [-L, L]$ and two variables are outside $[-L, L] \times [-L, L] \times [-L, L]$ (Sectors from 97 to 120).
4) All variables are outside $[-L, L] \times [-L, L] \times [-L, L]$ (Sectors from 121 to 128).

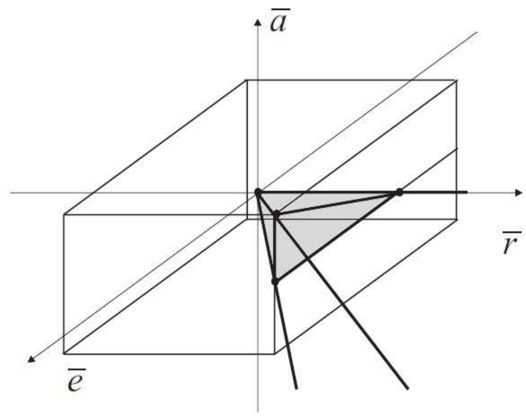


Fig. 5. Sector 1 in a cube.

For category 1), we consider the input–output relationship of Sector 1, which is the same as Cell$(1, 1, 1)$ in which the input variables satisfy the following ranges:

$$0 \le \bar{r} \le L, \; 0 \le \bar{e} \le L, \; 0 \le \bar{a} \le L,$$

TABLE I

A SECTOR IN THE CUBE IS MATCHED TO THE CORRESPONDING CELL IN THE TABLE (SECTORS FROM 1 TO 64). ('S' AND 'C' MEAN SECTOR AND CELL, RESPECTIVELY.)

S	C	S	C	S	C	S	C
1	(1,1,1)	17	(1,2,2)	33	(6,4,4)	49	(9,1,9)
2	(1,2,1)	18	(2,2,2)	34	(6,4,3)	50	(9,2,9)
3	(8,3,1)	19	(3,2,3)	35	(7,4,2)	51	(16,3,9)
4	(8,4,1)	20	(4,2,3)	36	(7,4,1)	52	(16,4,9)
5	(8,5,8)	21	(5,3,3)	37	(7,5,8)	53	(16,5,16)
6	(8,6,8)	22	(6,3,3)	38	(7,5,7)	54	(16,6,160)
7	(1,7,8)	23	(7,3,2)	39	(6,5,6)	55	(9,7,16)
8	(1,8,8)	24	(8,3,2)	40	(6,5,5)	56	(9,8,16)
9	(2,1,1)	25	(5,4,4)	41	(5,6,6)	57	(10,9,1)
10	(2,1,2)	26	(5,3,4)	42	(6,6,6)	58	(10,9,2)
11	(3,1,3)	27	(4,2,4)	43	(7,6,7)	59	(11,9,3)
12	(3,1,4)	28	(4,1,4)	44	(8,6,7)	60	(11,9,4)
13	(3,8,5)	29	(4,8,5)	45	(1,7,7)	61	(11,16,5)
14	(3,8,6)	30	(4,7,5)	46	(2,7,7)	62	(11,160,6)
15	(2,8,7)	31	(5,6,5)	47	(3,7,6)	63	(10,16,7)
16	(2,8,8)	32	(5,5,5)	48	(4,7,6)	64	(10,16,8)

TABLE II

A SECTOR IN THE CUBE IS MATCHED TO THE CORRESPONDING CELL IN THE TABLE (SECTORS FROM 65 TO 128). ('S' AND 'C' MEAN SECTOR AND CELL, RESPECTIVELY.)

S	C	S	C	S	C	S	C
65	(1,10,10)	81	(14,13,4)	97	(9,10,17)	113	(14,18,11)
66	(2,10,10)	82	(14,13,3)	98	(16,11,17)	114	(15,18,10)
67	(3,10,11)	83	(15,13,2)	99	(20,12,9)	115	(13,14,19)
68	(4,10,11)	84	(15,13,1)	100	(20,13,16)	116	(12,15,19)
69	(5,11,11)	85	(15,12,8)	101	(16,14,20)	117	(19,13,13)
70	(6,11,11)	86	(15,12,7)	102	(9,15,20)	118	(19,12,12)
71	(7,11,10)	87	(14,12,6)	103	(17,16,16)	119	(14,19,14)
72	(8,11,10)	88	(14,12,5)	104	(17,9,9)	120	(15,19,15)
73	(13,4,13)	89	(5,14,14)	105	(10,17,10)	121	(17,17,17)
74	(13,3,13)	90	(6,14,14)	106	(11,17,11)	122	(20,18,17)
75	(12,2,13)	91	(7,14,15)	107	(18,9,12)	123	(20,19,20)
76	(12,1,13)	92	(8,14,15)	108	(18,16,13)	124	(17,20,20)
77	(12,8,12)	93	(1,15,15)	109	(11,20,14)	125	(18,20,19)
78	(12,7,12)	94	(2,15,15)	110	(10,20,15)	126	(18,17,18)
79	(13,6,12)	95	(3,15,14)	111	(12,10,18)	127	(19,18,18)
80	(13,5,12)	96	(4,15,14)	112	(13,11,18)	128	(19,19,19)

and

$$\bar{e} \leq \bar{r},\ \bar{a} \leq \bar{e},\ \bar{a} \leq \bar{r},$$

that is, $\bar{r} \geq \bar{e} \geq \bar{a}$. From these three inequalities, we can derive the membership functions that satisfy the following inequalities in Sector 1:

$$\mu_{RN} \leq \mu_{EN} \leq \mu_{AN} \leq \mu_{AP} \leq \mu_{EP} \leq \mu_{RP}. \tag{3}$$

From (1) and (3), the MIN membership functions (μ_i for $i = 1, \cdots, 8$) for the eight rules in Sector 1 are:

$$\mu_i = \{\mu_{RN}, \mu_{RN}, \mu_{EN}, \mu_{EN}, \mu_{RN}, \mu_{RN}, \mu_{AN}, \mu_{AP}\}.$$

For category 1), we consider the input–output relationship of Sector 1, which is the same as Cell$(1,1,1)$. By using (2), $K_P(\bar{r},\bar{e},\bar{a})$, $K_I(\bar{r},\bar{e},\bar{a})$ and $K_D(\bar{r},\bar{e},\bar{a})$ of the controller in Sector 1 can then be expressed as

$$\begin{aligned} K_P(\bar{r},\bar{e},\bar{a}) &= \frac{(L-\bar{r})(\alpha_1+\alpha_2+\alpha_5+\alpha_6)}{-4\bar{r}-2\bar{e}+8L} \\ &+ \frac{(L-\bar{e})(\alpha_3+\alpha_4)}{-4\bar{r}-2\bar{e}+8L} \\ &+ \frac{(L-\bar{a})\alpha_7+(L+\bar{a})\alpha_8}{-4\bar{r}-2\bar{e}+8L}, \\ K_I(\bar{r},\bar{e},\bar{a}) &= \frac{(L-\bar{r})(\beta_1+\beta_2+\beta_5+\beta_6)}{-4\bar{r}-2\bar{e}+8L} \\ &+ \frac{(L-\bar{e})(\beta_3+\beta_4)}{-4\bar{r}-2\bar{e}+8L} \\ &+ \frac{(L-\bar{a})\beta_7+(L+\bar{a})\beta_8}{-4\bar{r}-2\bar{e}+8L}, \\ K_D(\bar{r},\bar{e},\bar{a}) &= \frac{(L-\bar{r})(\gamma_1+\gamma_2+\gamma_5+\gamma_6)}{-4\bar{r}-2\bar{e}+8L} \\ &+ \frac{(L-\bar{e})(\gamma_3+\gamma_4)}{-4\bar{r}-2\bar{e}+8L} \\ &+ \frac{(L-\bar{a})\gamma_7+(L+\bar{a})\gamma_8}{-4\bar{r}-2\bar{e}+8L}. \end{aligned}$$

With $K_P(\bar{r},\bar{e},\bar{a})$, $K_I(\bar{r},\bar{e},\bar{a})$ and $K_D(\bar{r},\bar{e},\bar{a})$, we can obtain the controller output. And the gains have same form. So we show only $K_P(\bar{r},\bar{e},\bar{a})$ for all sectors: Table III is for sectors from 1 to 48, Table IV is for sectors from 49 to 96 and Table V is for sectors from 97 to 128.

TABLE III

$K_P(\bar{r},\bar{e},\bar{a})$ OF SECTORS FROM 1 TO 48 (CATEGORY 1)).

Sector	$K_p\,(\bar{r},\bar{e},\bar{a})$
1, 8	$\frac{(L-\bar{r})(\alpha_1+\alpha_2+\alpha_5+\alpha_6)+(L-\bar{e})(\alpha_3+\alpha_4)+(L-\bar{a})\alpha_7+(L+\bar{a})\alpha_8}{-4\bar{r}-2\bar{e}+8L}$
2, 3	$\frac{(L-\bar{r})(\alpha_1+\alpha_2+\alpha_5+\alpha_6)+(L-\bar{a})(\alpha_3+\alpha_7)+(L-\bar{e})\alpha_4+(L+\bar{e})\alpha_8}{-4\bar{r}-2\bar{a}+8L}$
4, 5	$\frac{(L-\bar{r})(\alpha_1+\alpha_2+\alpha_5+\alpha_6)+(L+\bar{e})(\alpha_7+\alpha_8)+(L-\bar{a})\alpha_3+(L+\bar{a})\alpha_4}{-4\bar{r}+2\bar{e}+8L}$
6, 7	$\frac{(L-\bar{r})(\alpha_1+\alpha_2+\alpha_5+\alpha_6)+(L+\bar{a})(\alpha_4+\alpha_8)+(L-\bar{e})\alpha_3+(L+\bar{e})\alpha_7}{-4\bar{r}+2\bar{a}+8L}$
9, 16	$\frac{(L-\bar{e})(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+(L-\bar{r})(\alpha_5+\alpha_6)+(L-\bar{a})\alpha_7+(L+\bar{a})\alpha_8}{-4\bar{e}-2\bar{r}+8L}$
10, 11	$\frac{(L-\bar{e})(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+(L-\bar{a})(\alpha_5+\alpha_7)+(L-\bar{r})\alpha_6+(L+\bar{r})\alpha_8}{-4\bar{e}-2\bar{a}+8L}$
12, 13	$\frac{(L-\bar{e})(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+(L+\bar{r})(\alpha_7+\alpha_8)+(L-\bar{a})\alpha_5+(L+\bar{a})\alpha_6}{-4\bar{e}+2\bar{r}+8L}$
14, 15	$\frac{(L-\bar{e})(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+(L+\bar{a})(\alpha_6+\alpha_8)+(L-\bar{r})\alpha_5+(L+\bar{r})\alpha_7}{-4\bar{e}+2\bar{a}+8L}$
17, 24	$\frac{(L-\bar{a})(\alpha_1+\alpha_3+\alpha_5+\alpha_7)+(L-\bar{r})(\alpha_2+\alpha_6)+(L-\bar{e})\alpha_4+(L+\bar{e})\alpha_8}{-4\bar{a}-2\bar{r}+8L}$
18, 19	$\frac{(L-\bar{a})(\alpha_1+\alpha_3+\alpha_5+\alpha_7)+(L-\bar{e})(\alpha_2+\alpha_4)+(L-\bar{r})\alpha_6+(L+\bar{r})\alpha_8}{-4\bar{a}-2\bar{e}+8L}$
20, 21	$\frac{(L-\bar{a})(\alpha_1+\alpha_3+\alpha_5+\alpha_7)+(L+\bar{r})(\alpha_4+\alpha_8)+(L-\bar{e})\alpha_2+(L+\bar{e})\alpha_6}{-4\bar{a}+2\bar{r}+8L}$
22, 23	$\frac{(L-\bar{a})(\alpha_1+\alpha_3+\alpha_5+\alpha_7)+(L+\bar{e})(\alpha_6+\alpha_8)+(L-\bar{r})\alpha_2+(L+\bar{r})\alpha_4}{-4\bar{a}+2\bar{e}+8L}$
25, 32	$\frac{(L+\bar{r})(\alpha_3+\alpha_4+\alpha_7+\alpha_8)+(L+\bar{e})(\alpha_5+\alpha_6)+(L-\bar{a})\alpha_1+(L+\bar{a})\alpha_2}{4\bar{r}+2\bar{e}+8L}$
26, 27	$\frac{(L+\bar{r})(\alpha_3+\alpha_4+\alpha_7+\alpha_8)+(L+\bar{a})(\alpha_2+\alpha_6)+(L-\bar{e})\alpha_1+(L+\bar{e})\alpha_5}{4\bar{r}+2\bar{a}+8L}$
28, 29	$\frac{(L+\bar{r})(\alpha_3+\alpha_4+\alpha_7+\alpha_8)+(L-\bar{e})(\alpha_1+\alpha_2)+(L-\bar{a})\alpha_5+(L+\bar{a})\alpha_6}{4\bar{r}-2\bar{e}+8L}$
30, 31	$\frac{(L+\bar{r})(\alpha_3+\alpha_4+\alpha_7+\alpha_8)+(L-\bar{a})(\alpha_1+\alpha_5)+(L-\bar{e})\alpha_2+(L+\bar{e})\alpha_6}{4\bar{r}-2\bar{a}+8L}$
33, 40	$\frac{(L+\bar{e})(\alpha_5+\alpha_6+\alpha_7+\alpha_8)+(L+\bar{r})(\alpha_3+\alpha_4)+(L-\bar{a})\alpha_1+(L+\bar{a})\alpha_2}{4\bar{e}+2\bar{r}+8L}$
34, 35	$\frac{(L+\bar{e})(\alpha_5+\alpha_6+\alpha_7+\alpha_8)+(L+\bar{a})(\alpha_2+\alpha_4)+(L-\bar{r})\alpha_1+(L+\bar{r})\alpha_3}{4\bar{e}+2\bar{a}+8L}$
36, 37	$\frac{(L+\bar{e})(\alpha_5+\alpha_6+\alpha_7+\alpha_8)+(L-\bar{r})(\alpha_1+\alpha_2)+(L-\bar{a})\alpha_3+(L+\bar{a})\alpha_4}{4\bar{e}-2\bar{r}+8L}$
38, 39	$\frac{(L+\bar{e})(\alpha_5+\alpha_6+\alpha_7+\alpha_8)+(L-\bar{a})(\alpha_1+\alpha_3)+(L-\bar{r})\alpha_2+(L+\bar{r})\alpha_4}{4\bar{e}-2\bar{a}+8L}$
41, 48	$\frac{(L+\bar{a})(\alpha_2+\alpha_4+\alpha_6+\alpha_8)+(L+\bar{r})(\alpha_3+\alpha_7)+(L-\bar{e})\alpha_1+(L+\bar{e})\alpha_5}{4\bar{a}+2\bar{r}+8L}$
42, 43	$\frac{(L+\bar{a})(\alpha_2+\alpha_4+\alpha_6+\alpha_8)+(L+\bar{e})(\alpha_5+\alpha_7)+(L-\bar{r})\alpha_1+(L+\bar{r})\alpha_3}{4\bar{a}+2\bar{e}+8L}$
44, 45	$\frac{(L+\bar{a})(\alpha_2+\alpha_4+\alpha_6+\alpha_8)+(L-\bar{r})(\alpha_1+\alpha_5)+(L-\bar{e})\alpha_3+(L+\bar{e})\alpha_7}{4\bar{a}-2\bar{r}+8L}$
46, 47	$\frac{(L+\bar{a})(\alpha_2+\alpha_4+\alpha_6+\alpha_8)+(L-\bar{e})(\alpha_1+\alpha_3)+(L-\bar{r})\alpha_5+(L+\bar{r})\alpha_7}{4\bar{a}-2\bar{e}+8L}$

TABLE IV
$K_P(\bar{r},\bar{e},\bar{a})$ OF SECTORS FROM 49 TO 96 (CATEGORY 2)).

Sector	$K_p\,(\bar{r},\bar{e},\bar{a})$
49, 56	$\frac{(L-\bar{e})(\alpha_3+\alpha_4)+(L-\bar{a})\alpha_7+(L+\bar{a})\alpha_8}{-2\bar{e}+4L}$
73, 80	$\frac{(L+\bar{e})(\alpha_5+\alpha_6)+(L-\bar{a})\alpha_1+(L+\bar{a})\alpha_2}{2\bar{e}+4L}$
50, 51	$\frac{(L-\bar{a})(\alpha_3+\alpha_7)+(L-\bar{e})\alpha_4+(L+\bar{e})\alpha_8}{-2\bar{a}+4L}$
74, 75	$\frac{(L+\bar{a})(\alpha_2+\alpha_6)+(L-\bar{e})\alpha_1+(L+\bar{e})\alpha_5}{2\bar{a}+4L}$
52, 53	$\frac{(L+\bar{e})(\alpha_7+\alpha_8)+(L-\bar{a})\alpha_3+(L+\bar{a})\alpha_4}{2\bar{e}+4L}$
76, 77	$\frac{(L-\bar{e})(\alpha_1+\alpha_2)+(L-\bar{a})\alpha_5+(L+\bar{a})\alpha_6}{-2\bar{e}+4L}$
54, 55	$\frac{(L+\bar{a})(\alpha_4+\alpha_8)+(L-\bar{e})\alpha_3+(L+\bar{e})\alpha_7}{2\bar{a}+4L}$
78, 79	$\frac{(L-\bar{a})(\alpha_1+\alpha_5)+(L-\bar{e})\alpha_2+(L+\bar{e})\alpha_6}{-2\bar{a}+4L}$
57, 64	$\frac{(L-\bar{r})(\alpha_5+\alpha_6)+(L-\bar{a})\alpha_7+(L+\bar{a})\alpha_8}{-2\bar{r}+4L}$
81, 88	$\frac{(L+\bar{r})(\alpha_3+\alpha_4)+(L-\bar{a})\alpha_1+(L+\bar{a})\alpha_2}{2\bar{r}+4L}$
58, 59	$\frac{(L-\bar{a})(\alpha_5+\alpha_7)+(L-\bar{r})\alpha_6+(L+\bar{r})\alpha_8}{-2\bar{a}+4L}$
82, 83	$\frac{(L+\bar{a})(\alpha_2+\alpha_4)+(L-\bar{r})\alpha_1+(L+\bar{r})\alpha_3}{2\bar{a}+4L}$
60, 61	$\frac{(L+\bar{r})(\alpha_7+\alpha_8)+(L-\bar{a})\alpha_5+(L+\bar{a})\alpha_6}{2\bar{r}+4L}$
84, 85	$\frac{(L-\bar{r})(\alpha_1+\alpha_2)+(L-\bar{a})\alpha_3+(L+\bar{a})\alpha_4}{-2\bar{r}+4L}$
62, 63	$\frac{(L+\bar{a})(\alpha_6+\alpha_8)+(L-\bar{r})\alpha_5+(L+\bar{r})\alpha_7}{2\bar{a}+4L}$
86, 87	$\frac{(L-\bar{a})(\alpha_1+\alpha_3)+(L-\bar{r})\alpha_2+(L+\bar{r})\alpha_4}{-2\bar{a}+4L}$
65, 72	$\frac{(L-\bar{r})(\alpha_2+\alpha_6)+(L-\bar{e})\alpha_4+(L+\bar{e})\alpha_8}{-2\bar{r}+4L}$
89, 96	$\frac{(L+\bar{r})(\alpha_3+\alpha_7)+(L-\bar{e})\alpha_1+(L+\bar{e})\alpha_5}{2\bar{r}+4L}$
66, 67	$\frac{(L-\bar{e})(\alpha_2+\alpha_4)+(L-\bar{r})\alpha_6+(L+\bar{r})\alpha_8}{-2\bar{e}+4L}$
90, 91	$\frac{(L+\bar{e})(\alpha_5+\alpha_7)+(L-\bar{r})\alpha_1+(L+\bar{r})\alpha_3}{2\bar{e}+4L}$
68, 69	$\frac{(L+\bar{r})(\alpha_4+\alpha_8)+(L-\bar{e})\alpha_2+(L+\bar{e})\alpha_6}{2\bar{r}+4L}$
92, 93	$\frac{(L-\bar{r})(\alpha_1+\alpha_5)+(L-\bar{e})\alpha_3+(L+\bar{e})\alpha_7}{-2\bar{r}+4L}$
70, 71	$\frac{(L+\bar{e})(\alpha_6+\alpha_8)+(L-\bar{r})\alpha_2+(L+\bar{r})\alpha_4}{2\bar{e}+4L}$
94, 95	$\frac{(L-\bar{e})(\alpha_1+\alpha_3)+(L-\bar{r})\alpha_5+(L+\bar{r})\alpha_7}{-2\bar{e}+4L}$

TABLE V
$K_P(\bar{r},\bar{e},\bar{a})$ OF SECTORS FROM 97 TO 128 (CATEGORY 3) AND 4)).

Sector	$K_p\,(\bar{r},\bar{e},\bar{a})$	Sector	$K_p\,(\bar{r},\bar{e},\bar{a})$
97, 98	$\frac{(L-\bar{e})\alpha_4+(L+\bar{e})\alpha_8}{2L}$	117, 118	$\frac{(L-\bar{a})\alpha_1+(L+\bar{a})\alpha_2}{2L}$
99, 100	$\frac{(L-\bar{a})\alpha_3+(L+\bar{a})\alpha_4}{2L}$	119, 120	$\frac{(L-\bar{r})\alpha_1+(L+\bar{r})\alpha_3}{2L}$
101, 102	$\frac{(L-\bar{e})\alpha_3+(L+\bar{e})\alpha_7}{2L}$	121	α_8
103, 104	$\frac{(L-\bar{a})\alpha_7+(L+\bar{a})\alpha_8}{2L}$	122	α_4
105, 106	$\frac{(L-\bar{r})\alpha_6+(L+\bar{r})\alpha_8}{2L}$	123	α_3
107, 108	$\frac{(L-\bar{a})\alpha_5+(L+\bar{a})\alpha_6}{2L}$	124	α_7
109, 110	$\frac{(L-\bar{r})\alpha_5+(L+\bar{r})\alpha_7}{2L}$	125	α_5
111, 112	$\frac{(L-\bar{e})\alpha_2+(L+\bar{e})\alpha_6}{2L}$	126	α_6
113, 114	$\frac{(L-\bar{r})\alpha_2+(L+\bar{r})\alpha_4}{2L}$	127	α_2
115, 116	$\frac{(L-\bar{e})\alpha_1+(L+\bar{e})\alpha_5}{2L}$	128	α_1

TABLE VI
K_p^{max} OF SECTORS FROM 1 TO 48 (CATEGORY 1)).

Sector	K_p^{max}
25, 26, 33, 34, 41, 42	$\frac{2\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}{2}$
21, 22, 31, 32, 39, 40	$\frac{\alpha_1+2\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}{2}$
5, 6, 35, 36, 43, 44	$\frac{\alpha_1+\alpha_2+2\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}{2}$
3, 4, 23, 24, 37, 38	$\frac{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}{2}$
13, 14, 27, 28, 47, 48	$\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}{2}$
11, 12, 19, 20, 29, 30	$\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+2\alpha_6+\alpha_7+\alpha_8}{2}$
8, 7, 15, 16, 45, 46	$\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+2\alpha_7+\alpha_8}{2}$
1, 2, 9, 10, 17, 18	$\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+2\alpha_8}{2}$

TABLE VII
K_p^{max} OF SECTORS FROM 49 TO 96 (CATEGORY 2)).

Sector	K_p^{max}	Sector	K_p^{max}
49, 50	$\frac{\alpha_3+\alpha_4+\alpha_7+2\alpha_8}{2}$	73, 74	$\frac{2\alpha_1+\alpha_2+\alpha_5+\alpha_6}{2}$
51, 52	$\frac{\alpha_3+2\alpha_4+\alpha_7+\alpha_8}{2}$	75, 76	$\frac{\alpha_1+\alpha_2+2\alpha_5+\alpha_6}{2}$
53, 54	$\frac{2\alpha_3+\alpha_4+\alpha_7+\alpha_8}{2}$	77, 78	$\frac{\alpha_1+\alpha_2+\alpha_5+2\alpha_6}{2}$
55, 56	$\frac{\alpha_3+\alpha_4+2\alpha_7+\alpha_8}{2}$	79, 80	$\frac{\alpha_1+2\alpha_2+\alpha_5+\alpha_6}{2}$
57, 58	$\frac{\alpha_5+\alpha_6+\alpha_7+2\alpha_8}{2}$	81, 82	$\frac{2\alpha_1+\alpha_2+\alpha_3+\alpha_4}{2}$
59, 60	$\frac{\alpha_5+2\alpha_6+\alpha_7+\alpha_8}{2}$	83, 84	$\frac{\alpha_1+\alpha_2+2\alpha_3+\alpha_4}{2}$
61, 62	$\frac{2\alpha_5+\alpha_6+\alpha_7+\alpha_8}{2}$	85, 86	$\frac{\alpha_1+\alpha_2+\alpha_3+2\alpha_4}{2}$
63, 64	$\frac{\alpha_5+\alpha_6+2\alpha_7+\alpha_8}{2}$	87, 88	$\frac{\alpha_1+2\alpha_2+\alpha_3+\alpha_4}{2}$
65, 66	$\frac{\alpha_2+\alpha_4+\alpha_6+2\alpha_8}{2}$	89, 90	$\frac{2\alpha_1+\alpha_3+\alpha_5+\alpha_7}{2}$
67, 68	$\frac{\alpha_2+\alpha_4+2\alpha_6+\alpha_8}{2}$	91, 92	$\frac{\alpha_1+2\alpha_3+\alpha_5+\alpha_7}{2}$
69, 70	$\frac{2\alpha_2+\alpha_4+\alpha_6+\alpha_8}{2}$	93, 94	$\frac{\alpha_1+\alpha_3+\alpha_5+2\alpha_7}{2}$
71, 72	$\frac{\alpha_2+2\alpha_4+\alpha_6+\alpha_8}{2}$	95, 96	$\frac{\alpha_1+\alpha_3+2\alpha_5+\alpha_7}{2}$

TABLE VIII
K_p^{max} OF SECTORS FROM 97 TO 128 (CATEGORY 3) AND 4)).

Sector	K_p^{max}	Sector	K_p^{max}
97	$\frac{\alpha_4+2\alpha_8}{2}$	105	$\frac{\alpha_6+2\alpha_8}{2}$
98	$\frac{2\alpha_4+\alpha_8}{2}$	106	$\frac{2\alpha_6+\alpha_8}{2}$
99	$\frac{\alpha_3+2\alpha_4}{2}$	107	$\frac{\alpha_5+2\alpha_6}{2}$
100	$\frac{2\alpha_3+\alpha_4}{2}$	108	$\frac{2\alpha_5+\alpha_6}{2}$
101	$\frac{2\alpha_3+\alpha_7}{2}$	109	$\frac{2\alpha_5+\alpha_7}{2}$
102	$\frac{\alpha_3+2\alpha_7}{2}$	110	$\frac{\alpha_5+2\alpha_7}{2}$
103	$\frac{2\alpha_7+\alpha_8}{2}$	111	$\frac{\alpha_2+2\alpha_6}{2}$
104	$\frac{\alpha_7+2\alpha_8}{2}$	112	$\frac{2\alpha_2+\alpha_6}{2}$
113	$\frac{2\alpha_2+\alpha_4}{2}$	121	α_8
114	$\frac{\alpha_2+2\alpha_4}{2}$	122	α_4
115	$\frac{2\alpha_1+\alpha_5}{2}$	123	α_3
116	$\frac{\alpha_1+2\alpha_5}{2}$	124	α_7
117	$\frac{2\alpha_1+\alpha_2}{2}$	125	α_5
118	$\frac{\alpha_1+2\alpha_2}{2}$	126	α_6
119	$\frac{2\alpha_1+\alpha_3}{2}$	127	α_2
120	$\frac{\alpha_1+2\alpha_3}{2}$	128	α_1

IV. STABILITY ANALYSIS OF TSK-TYPE FULL–SCALE FUZZY PID CONTROL SYSTEM

In this section, we prove that the control system formed using the proposed controller becomes BIBO stable whenever the plant satisfies finite–gain ℓ–stability.

If N is nonlinear and its output is $N(u(nT))$, the fuzzy PID control system (Fig. 6) can be expressed as

$$
\begin{aligned}
e_1(nT) &= e(nT),\\
e_2(nT) &= u(nT),\\
u_1(nT) &= SP,\\
u_2(nT) &= u((n-1)T),\\
S_1(e_1(nT)) &= \Delta u(nT),\\
S_2(e_2(nT)) &= N(e_2(nT)).
\end{aligned}
$$

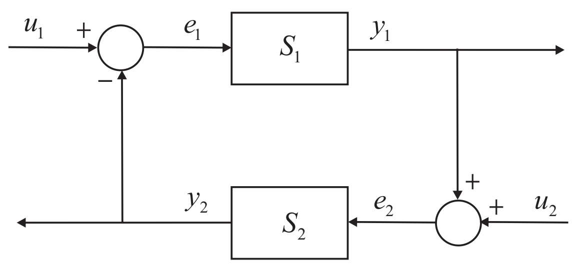


Fig. 6. Feedback System.

If the input variables are contained in Sector 1 or 8, then

$$\begin{aligned}
&\|S_1(e_1(nT))\| \\
&= \|K_P\bar{r}(nT) + K_I\bar{e}(nT) + K_D\bar{a}(nT)\| \\
&= \|K_P\left(\bar{e}(nT) - \bar{e}((n-1)T)\right) + K_I\bar{e}(nT) \\
&\quad + K_D\left(\bar{e}(nT) - 2\bar{e}((n-1)T) + \bar{e}((n-2)T)\right)\| \\
&\leq (K_rK_P + K_eK_I + K_aK_D)\,|e(nT)| \\
&\quad + (K_PK_r + 2K_DK_a)\,|e((n-1)T)| + K_DK_a\,|e((n-2)T)| \\
&\leq (K_rK_P + K_eK_I + K_aK_D)\,|e(nT)| \\
&\quad + (K_PK_r + 2K_DK_a)M_{e1} + K_DK_aM_{e2}
\end{aligned}$$

where $M_{e1} = \sup_{n\geq 1} |e\left((n-1)\,T\right)|$ and $M_{e2} = \sup_{n\geq 2} |e\left((n-2)\,T\right)|$.

Because $0 \leq \bar{r}, \bar{e}, \bar{a} \leq L$,

$$\begin{aligned}
&K_rK_P + K_eK_I + K_aK_D \\
&= \frac{1}{-4\bar{r} - 2\bar{e} + 8L} \cdot \{K_r((L-\bar{r})(\alpha_1 + \alpha_2 + \alpha_5 + \alpha_6) \\
&\quad + (L-\bar{e})(\alpha_3 + \alpha_4) + (L-\bar{a})\alpha_7 + (L+\bar{a})\alpha_8) \\
&\quad + K_e((L-\bar{r})(\beta_1 + \beta_2 + \beta_5 + \beta_6) \\
&\quad + (L-\bar{e})(\beta_3 + \beta_4) + (L-\bar{a})\beta_7 + (L+\bar{a})\beta_8) \\
&\quad + K_a((L-\bar{r})(\gamma_1 + \gamma_2 + \gamma_5 + \gamma_6) \\
&\quad + (L-\bar{e})(\gamma_3 + \gamma_4) + (L-\bar{a})\gamma_7 + (L+\bar{a})\gamma_8)\} \\
&\leq K_rK_P^{\max} + K_eK_I^{\max} + K_aK_D^{\max}
\end{aligned}$$

where

$$\begin{aligned}
K_P^{\max} &= \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + 2\alpha_8}{2}, \\
K_I^{\max} &= \frac{\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + 2\beta_8}{2}, \\
K_D^{\max} &= \frac{\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 + \gamma_7 + 2\gamma_8}{2}.
\end{aligned}$$

The same procedure can be repeated to obtain K_P^{max} for the other sectors (Table VI, VII and VIII). K_I^{max} and K_D^{max} are of the same form as K_P^{max}.

Now

$$\|S_2\left(e_2\left(nT\right)\right)\| \leq \|N\| \cdot |e_2\left(nT\right)|$$

where $\|N\| < \infty$. N represents a nonlinear/time-varying system in the form of a nonlinear differential equation, and $\|N\|$ is the maximum of system gain. Then, by using the Small–Gain Theorem, the control system becomes BIBO stable if

$$K^{max} \cdot \|N\| < 1$$

where

$$K^{\max} = K_r \sum_{i=1}^{8} \alpha_i + K_e \sum_{i=1}^{8} \beta_i + K_a \sum_{i=1}^{8} \gamma_i. \quad (4)$$

Theorem 1. *The sufficient conditions for a TSK–type full–scale fuzzy PID controller to stabilize the given plant N in the BIBO sense are*

1) $\|N\| < \infty$,
2) $K^{max} \cdot \|N\| < 1$

where K^{max} is given in (4).

Note here that L is not related to the BIBO stability in the TSK–type full–scale fuzzy PID controller case.

V. Simulation Results

In this section, we performed a computer simulation using MATLAB to compare rising time, overshoot and settling time of the TSK–type full–scale fuzzy PID controlled system to those of the conventional PID controlled system.

The control parameters were determined by trial and error. In this simulation, the set point of system output was 1, and the sampling period T was 0.01s. For Step 1, to obtain reasonably good rising time, overshoot and settling time, we first roughly tuned K_r and K_e. Then, we fine–tuned K_r, K_e, α_i, β_i, and γ_i for $i = 1, \cdots, 8$ to achieve fast response and stability. Finally, we tuned K_a to reduce oscillation. We applied the evaluation process to the first–order model with a time–delay. The control parameters of the TSK–type full–scale fuzzy PID controller were selected as $L = 0.5$, $K_r = 7$, $K_e = 0.006$, $K_a = 0.01$, $\alpha_1 = 0.01$, $\alpha_2 = 0.02$, $\alpha_3 = 0.03$, $\alpha_4 = 0.04$, $\alpha_5 = 0.05$, $\alpha_6 = 0.06$, $\alpha_7 = 0.07$, $\alpha_8 = 0.09$, $\beta_1 = 0.1$,

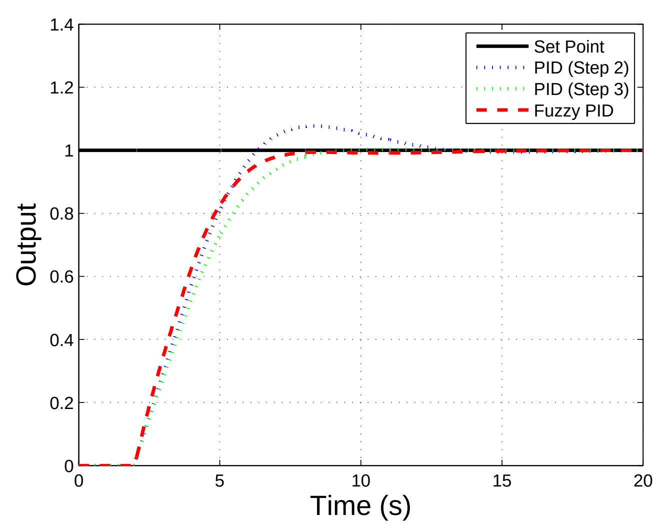


Fig. 7. Comparison between the conventional PID controller and the TSK–type full–scale fuzzy PID controller in the first–order model with a time–delay.

$\beta_2 = 0.1$, $\beta_3 = 0.4$, $\beta_4 = 0.2$, $\beta_5 = 0.5$, $\beta_6 = 0.7$, $\beta_7 = 0.7$, $\beta_8 = 0.5$, $\gamma_1 = 0.01$, $\gamma_2 = 0.01$, $\gamma_3 = 0.03$, $\gamma_4 = 0.03$, $\gamma_5 = 0.05$, $\gamma_6 = 0.05$, $\gamma_7 = 0.07$, and $\gamma_8 = 0.08$ for Step 1; the gains of the conventional PID controller were set to P–gain $= 0.29$, I–gain $= 0.0028$ and D–gain $= 1$ for Step 2 and P–gain $= 0.29$, I–gain $= 0.0024$ and D–gain $= 1$ for Step 3. The TSK–type full–scale fuzzy PID controller showed small overshoot and short settling time in Step 2, and shorter rising time in Step 3 compared to those of the conventional PID controller (Fig. 7).

VI. Conclusions

In this paper, we introduced a TSK–type full–scale fuzzy PID controller structure and analyzed its stability. We employed only two fuzzy sets for each of the three input variables and adopted singleton fuzzy sets for output. Using the Small–Gain Theorem, we derived conditions for BIBO stability from the input–output relationship of the controller. However, the fuzzy PID controllers have more parameters to adjust than those of the conventional one. In the future, parameter tuning techniques, the generalization of the results to the case of more membership functions per variable, and the use of the MIN operator instead of the PRODUCT operator will be studied.

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