Sufficient Conditions for Monotonically Constrained Functional-type SIRMs Connected Fuzzy Systems

Jinwook Kim and Jin S. Lee

Abstract— **Monotonic input-output relationship is common in many physical systems. When modeling a physical system whose input-output relationship is monotonic, it is desired for a model to have the monotonicity. In this paper, we construct a model with single input rule modules (SIRMs) connected fuzzy system to solve "the curse of dimensionality", and propose sufficient conditions for the constructed fuzzy system to have monotonic input-output relationship. The conditions are obtained by restricting the first derivative of the fuzzy system to be nonnegative. The derived conditions can be classified into two parts: the condition on the consequent part parameters of fuzzy rules and the condition on the input membership functions. The simulation results show the validity of the derived conditions.**

I. Introduction

For many real world engineering problems, we often encounter monotonic input-output relationship. An example is the automated current control of magnet cranes in the steel plate yard [1]. In the yard, the steel plates are stacked in the layered form and an expert operates the crane to lift the desired number of plates and to move them to the destination. When we model an automatic current supply system, the input to the system is the number of plates to lift and the output is the amount of current supplied by the expert. We do not know the exact formula between its input and output, but we know clearly that the output is monotonically increasing with respect to its input: the expert supplies more current to lift more steel plates.

In recent years, several researchers have reported the results to improve the performance of the system that satisfies the monotonicity between its input and output. Wu et al. [2], [3] proposed a fuzzy controller based on the mean-of-inversion defuzzification method to deal with the monotonic functions. The work was motivated by the application of image compression. In the image compression process, the relationship between input parameter quality and the distortion ratio average gray scale error is monotonic. Zhao et al. [4] presented a monotonic rule base and a sufficient condition for the monotonicity of the fuzzy control algorithm. Lindskog et al. [5] considered the situation where the steady-state gain curve of a nonlinear dynamic system is monotonic and identified the system with a fuzzy system that ensures the monotonicity between its input and output. Won et al. [6] derived general sufficient conditions for the monotonicity of a Takagi-Sugeno-Kang fuzzy system by restricting the first derivative of a fuzzy output to be nonnegative. Kouikoglou and Phillis [7] presented the parameter conditions of a hierarchical fuzzy system to preserve the monotonicity.

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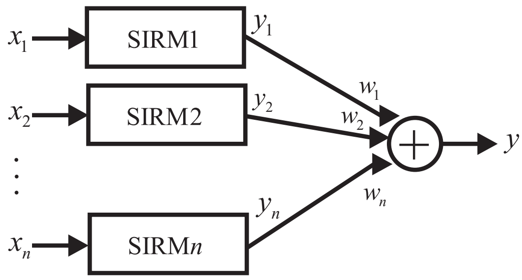

Fig. 1. A diagram of SIRMs connected fuzzy systems.

In these results except for [7], they did not consider "the curse of dimensionality" [8] which could be the most serious problem in building a fuzzy system in the real world: the number of rules increases exponentially with the number of input variables, and the number of tuning parameters also increases exponentially with the number of input variables. Yubazaki et al. [9] proposed the single input rule modules (SIRMs) connected fuzzy systems (Fig. 1) to solve this problem and demonstrated the significant reduction in the required fuzzy rules and good performances in many examples [10]-[12]. Seki et al. [13] proposed a functional-type SIRMs connected fuzzy system and, in [13] and [14], showed that the increasing ordering of input fuzzy sets in the SIRMs connected fuzzy system cannot guarantee the monotonicity of the fuzzy system even if the consequent parts are arranged by the increasing order, and proposed additional constraints on the input membership functions to preserve its monotonicity. However, we found that Seki's results cannot preserve the monotonicity of the fuzzy system.

To take the monotonicity and "the curse of dimensionality" into account, we derive in this paper sufficient conditions for a functional-type SIRMs connected fuzzy system to be monotonic. First, we show by using an example that the functional-type SIRMs connected fuzzy system constructed with Seki's results cannot preserve its monotonicity. To solve the problem, we derive sufficient conditions for the monotonicity by setting the first derivative of the fuzzy output to be nonnegative. These conditions consist of the conditions on the consequent part parameters and the condition on the input membership functions. These conditions are different from Seki's results [14]: there is one more condition on the consequent part parameters and the condition on the

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membership functions is less conservative.

This paper is organized as follows: In Section II, we introduce the basic concepts about fuzzy sets and functional-type SIRMs connected fuzzy systems. In Section III, we derive sufficient conditions for functional-type SIRMs connected fuzzy systems to be monotonic. In Section IV, we demonstrate the derived conditions through simulation examples. In Section V, we make conclusion.

II. Basic Definitions and Preliminaries

A monotonic function can be monotonically increasing or decreasing, but without loss of generality, only the monotonically increasing case is considered. Formally, the monotonic function is given as follows.

Definition 1: *(Monotonic Function)*[6] Let $\mathbf{x} = (x_1, \cdots, x_n)^T \in U = [\alpha_1, \beta_1] \times \cdots \times [\alpha_n, \beta_n] \subset \mathbb{R}^n$ be an input vector of the function $y = f(\mathbf{x}) \in V \subset \mathbb{R}$. Then, $f : U \mapsto V$ is said to be *monotonically increasing* if $f(\mathbf{x}^1) \leq f(\mathbf{x}^2)$ whenever $\mathbf{x}^1 \leq \mathbf{x}^2$ where $\mathbf{x}^1 = (x_1^1, \cdots, x_n^1)^T$ and $\mathbf{x}^2 = (x_1^2, \cdots, x_n^2)^T$.

By Definition 1, we can state that the given function is monotonic with respect to its input if the function is continuously differentiable and the partial derivatives of the function are nonnegative.

Definition 2: *(Pseudo-Trapezoid (PT) Membership Function)*[15] For a, b, c, d, and $U = [\alpha, \beta]$ such that $\alpha \leq a \leq b \leq c \leq d \leq \beta$, PT membership function over U is given by

$$\mu(x; a, b, c, d) = \begin{cases} I(x), \ x \in [a, b) \\ 1, \ x \in [b, c] \\ D(x), \ x \in (c, d] \\ 0, \ x \in U - (a, d) \end{cases}$$

where $I(x)$ and $D(x)$ are monotonically increasing and decreasing functions of x, respectively, such that $0 \leq I(x) \leq 1$ and $0 \leq D(x) \leq 1$.

Definition 3: *(Completeness)*[15] Fuzzy sets $A^1, \cdots, A^n$ in $U \subset \mathbb{R}$ are said to be *complete* on U if for any $x \in U$, there exists a fuzzy set A^l such that $\mu^l(x) > 0$.

Definition 4: *(Consistency)*[15] Fuzzy sets $A^1, \cdots, A^n$ in $U \subset \mathbb{R}$ are said to be *consistent* on U if whenever $\mu^l(x) = 1$ for some $x \in U$, then $\mu^k(x) = 0$ for all $l \neq k$.

Seki et al. [13] proposed the functional-type SIRMs connected fuzzy systems where the consequent part of each SIRM is a function of inputs.

Assume that the fuzzy rule for the jth SIRM consists of M_j rules as follows:

$$\text{SIRM}j: \ \{\text{IF } x_j \text{ is } A_j^{l_j} \text{ THEN } y_j \text{ is } c_j^{l_j} x_j + c_0^{l_j}\}.$$

where j for $1 \leq j \leq n$ is the index of input, $A_j^{l_j}$ for $1 \leq l_j \leq M_j$ is the input fuzzy set, and $c_j^{l_j}$ and $c_0^{l_j}$ are real constants in the consequent part.

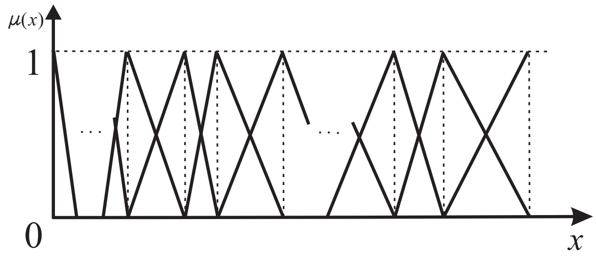

Fig. 2. The arrangement of input membership functions in Seki's results.

If we use singleton fuzzifier and center average defuzzifier for each module, then the fuzzy output is given as

$$f(\mathbf{x}) = \sum_{i=1}^{n} w_i y_i = \sum_{i=1}^{n} w_i \frac{\sum_{l_i=1}^{M_i} \left(c_i^{l_i} x_i + c_0^{l_i}\right) \mu_i^{l_i}(x_i)}{\sum_{l_i=1}^{M_i} \mu_i^{l_i}(x_i)} \quad (1)$$

where $\mu_i^{l_i}(x_i)$ is a membership function for fuzzy set $A_i^{l_i}$.

III. Monotonically Constrained Functional-type SIRMs Connected Fuzzy Systems

In [13], Seki et al. proved that a functional-type SIRMs connected fuzzy system is monotonically increasing with respect to its input if

1) $f_j^1(x_j) \leq \cdots \leq f_j^{M_j}(x_j)$
2) triangular membership functions are arranged as Fig. 2

where $f_j^{l_j}(x_j) = c_j^{l_j} x_j + c_0^{l_j}$. In the following example, we will show that the above conditions cannot guarantee the monotonicity of the constructed system.

Example 1: Assume that the two-input functional-type SIRMs connected fuzzy system has the following rule modules.

$$\text{SIRM1}: \begin{cases} \text{IF } x_1 = A_1^1 \text{ THEN } y_1 = -10x_1 - 1 \\ \text{IF } x_1 = A_1^2 \text{ THEN } y_1 = 1.5x_1 + 1 \end{cases}$$

$$\text{SIRM2}: \begin{cases} \text{IF } x_2 = A_2^1 \text{ THEN } y_2 = -5x_2 + 1 \\ \text{IF } x_2 = A_2^2 \text{ THEN } y_2 = 2x_2 + 2 \end{cases}$$

where $\mathbf{x} \in U = [0, 1] \times [0, 1]$ and A_i^j for $1 \leq i, j \leq 2$ are characterized by the triangular membership functions (Fig. 3) that satisfy Seki's second condition. And the consequent parts of SIRM1 and SIRM2 also satisfy Seki's first condition in U. Without loss of generality, we set $w_1 = w_2 = 1$.

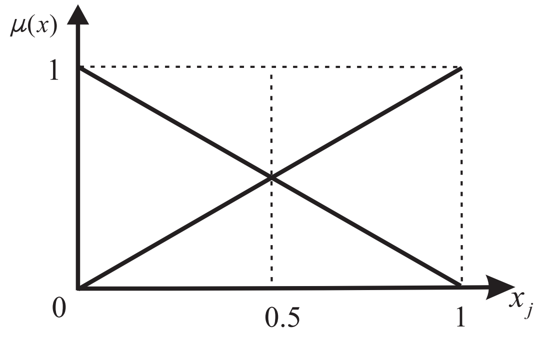

Fig. 3. Triangular membership functions in Example 1 ($j = 1, 2$).

- Given inputs $\mathbf{x} = (0, 0)$,

$$
\begin{aligned}
y =& w_1 \frac{f_1^1(x_1)\mu_1^1(x_1) + f_1^2(x_1)\mu_1^2(x_1)}{\mu_1^1(x_1) + \mu_1^2(x_1)} \\
&+ w_2 \frac{f_2^1(x_2)\mu_2^1(x_2) + f_2^2(x_2)\mu_2^2(x_2)}{\mu_2^1(x_2) + \mu_2^2(x_2)} \\
=& \frac{-1+0}{1+0} + \frac{1+0}{1+0} \\
=& 0.
\end{aligned}
$$

- Given inputs $\mathbf{x} = (0.4, 0.4)$,

$$
\begin{aligned}
y &= \frac{(-5)\cdot 0.6 + 1.6 \cdot 0.4}{0.6 + 0.4} + \frac{(-1)\cdot 0.6 + 2.8 \cdot 0.4}{0.6 + 0.4} \\
&= -3 + 0.64 - 0.6 + 1.12 \\
&= -1.84.
\end{aligned}
$$

From the above results, the output of the fuzzy system decreases from 0 to -1.84 although the input increases from $(0, 0)$ to $(0.4, 0.4)$. Here we show that, even if Seki's conditions are satisfied, the output of the functional-type SIRMs fuzzy system can decrease with respect to its input. For the subsequent development, we adopt the following two assumptions on the membership functions:
For $U = U_1 \times \cdots \times U_n \subset \mathbb{R}^n$,

1) Every membership function is continuous in U.
2) Every membership function is differentiable except for some finite points in U.

The popular membership functions such that triangular membership function, trapezoid membership function, bell-shaped membership function and so on satisfy the above assumptions.

Let $\bar{U} = \bar{U}_1 \times \cdots \times \bar{U}_n$ be defined as $U_j - \Gamma_j$ where Γ_j is the set of the non-differentiable points in U_j for $1 \le j \le n$.

Theorem 1: The functional-type SIRMs connected fuzzy system as given in Eq. (1) is monotonically increasing with respect to $x_j \in \bar{U}_j$ for $1 \le j \le n$ if the following conditions are satisfied:

1) $c_j^l \ge 0$ for all $1 \le l \le M_j$.
2) $c_j^p x_j + c_0^p \le c_j^q x_j + c_0^q$ for all $1 \le p < q \le M_j$.
3) $\left(d\mu_j^p(x_j)/dx_j\right)\mu_j^q(x_j) \le \mu_j^p(x_j)\left(d\mu_j^q(x_j)/dx_j\right)$ for all $1 \le p < q \le M_j$.

Proof: From Eq. (1),

$$
f(\mathbf{x}) = w_1 y_1 + \cdots + w_n y_n,
$$

and $\partial y/\partial x_j = w_j(\partial y_j/\partial x_j)$. Because $w_j > 0$, we only consider $\partial y_j/\partial x_j$. By using the formula $(f/g)' = (f'g - fg')/g^2$, we can obtain the following equation:

$$
\begin{aligned}
\frac{dy_j}{dx_j} =& \frac{1}{\sigma^2} \times \left[\sum_{p=1}^{M_j} \left(c_j^p \mu_j^p(x_j) + \left(c_j^p x_j + c_0^p\right) \frac{d\mu_j^p(x_j)}{dx_j} \right) \times \right. \\
& \left. \sum_{q=1}^{M_j} \mu_j^q(x_j) - \sum_{p=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \mu_j^p(x_j) \sum_{q=1}^{M_j} \frac{d\mu_j^q(x_j)}{dx_j} \right] \\
=& \frac{1}{\sigma^2} \times \left[\sum_{p=1}^{M_j} c_j^p \mu_j^p(x_j) \sum_{q=1}^{M_j} \mu_j^q(x_j) \right. \\
& + \sum_{p=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \frac{d\mu_j^p(x_j)}{dx_j} \sum_{q=1}^{M_j} \mu_j^q(x_j) \\
& \left. - \sum_{p=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \mu_j^p(x_j) \sum_{q=1}^{M_j} \frac{d\mu_j^q(x_j)}{dx_j} \right] \\
=& \frac{1}{\sigma^2} \times \left[\sum_{p=1}^{M_j} \sum_{q=1}^{M_j} c_j^p \mu_j^p(x_j) \mu_j^q(x_j) \right. \\
& + \sum_{p=1}^{M_j} \sum_{q=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \frac{d\mu_j^p(x_j)}{dx_j} \mu_j^q(x_j) \\
& \left. - \sum_{p=1}^{M_j} \sum_{q=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \mu_j^p(x_j) \frac{d\mu_j^q(x_j)}{dx_j} \right] \qquad (2)
\end{aligned}
$$

where $\sigma = \sum_{l=1}^{M_j} \mu_j^l(x_j)$. Since a membership value is non-negative and $c_j^l \ge 0$ (condition 1)) for $1 \le l \le M_j$, Eq. (2) becomes

$$
\begin{aligned}
\frac{dy_j}{dx_j} \ge& \frac{1}{\sigma^2} \times \left[\sum_{p=1}^{M_j} \sum_{q=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \frac{d\mu_j^p(x_j)}{dx_j} \mu_j^q(x_j) \right. \\
& \left. - \sum_{p=1}^{M_j} \sum_{q=1}^{M_j} \left(c_j^p x_j + c_0^p\right) \mu_j^p(x_j) \frac{d\mu_j^q(x_j)}{dx_j} \right] \\
=& \frac{1}{\sigma^2} \times \sum_{p=1}^{M_j} \sum_{q=1}^{M_j} \left(\left(c_j^p x_j + c_0^p - c_j^q x_j - c_0^q\right) \right. \\
& \left. \times \frac{d\mu_j^p(x_j)}{dx_j} \mu_j^q(x_j) \right) \\
=& \frac{1}{\sigma^2} \times \sum_{p=1}^{M_j - 1} \sum_{q=p+1}^{M_j} \left[\left(c_j^p x_j + c_0^p - c_j^q x_j - c_0^q\right) \right. \\
& \left. \times \left(\frac{d\mu_j^p(x_j)}{dx_j} \mu_j^q(x_j) - \mu_j^p(x_j) \frac{d\mu_j^q(x_j)}{dx_j} \right) \right]. \qquad (3)
\end{aligned}
$$

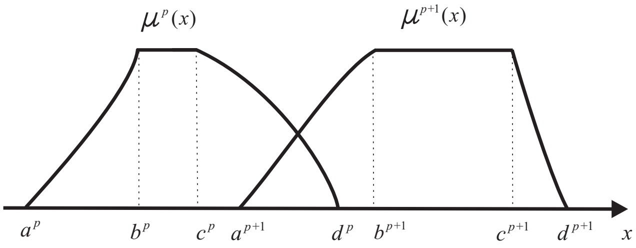

Fig. 4. Complete and consistent fuzzy sets characterized by PT membership functions.

Consequently, if $c_j^p x_j + c_0^p \le c_j^q x_j + c_0^q$ and $(d\mu_j^p(x_j)/dx_j)\mu_j^q(x_j) \le \mu_j^p(x_j)(d\mu_j^q(x_j)/dx_j)$ for all $x_j \in U_j$ and $1 \le p < q \le M_j$ in Eq. (3) are satisfied, then $dy_j/dx_j \ge 0$. And $dy/dx_j \ge 0$ can be achieved from $dy_j/dx_j \ge 0$. ■

The derived conditions in Theorem 1 can be classified into two parts: Conditions 1) and 2) are for consequent part parameters and condition 3) is for the input membership functions. Compared with Seki's conditions, Theorem 1 contains one more condition $c_j^l \ge 0$ which means that each consequent part of rules also must be monotonically increasing to obtain the monotonicity, i.e., $df_j^l(x_j)/dx_j \ge 0$. And the proof shows that if all of the SIRMs are monotonically increasing with respect to input, the overall output of functional-type SIRMs fuzzy system is monotonically increasing with respect to its input while satisfying condition 3).

With condition 3) of Theorem 1, we derive the useful lemmas for the monotonicity.

Lemma 1: [16] The set of PT membership functions built with complete and consistent fuzzy sets satisfies $(d\mu^p(x)/dx)\mu^q(x) \le \mu^p(x)(d\mu^q(x)/dx)$ for all $1 \le p < q \le M$.

Proof: PT membership functions with complete and consistent fuzzy sets satisfy $c^p \le a^{p+1} < d^p \le b^{p+1}$ for $1 \le p \le M-1$ as shown in Fig. 4.

If $x \in U - (a^p, d^{p+1})$, then $(d\mu^p(x)/dx)\mu^{p+1}(x) - \mu^p(x)(d\mu^{p+1}(x)/dx) = 0$ because $\mu_{A^p}(x) = 0$ and $\mu^{p+1}(x) = 0$. If $x \in (a^p, a^{p+1}]$, then $(d\mu^p(x)/dx)\mu^{p+1}(x) - \mu^p(x)(d\mu^{p+1}(x)/dx) = 0$ because $\mu^{p+1}(x) = 0$ and $d\mu^{p+1}(x)/dx = 0$. If $x \in [d^p, d^{p+1})$, then $(d\mu^p(x)/dx)\mu^{p+1}(x) - \mu^p(x)(d\mu^{p+1}(x)/dx) = 0$ because $\mu^p(x) = 0$ and $d\mu^p(x)/dx = 0$. If $x \in (a^{p+1}, d^p)$, then $(d\mu^p(x)/dx)\mu^{p+1}(x) - \mu^p(x)(d\mu^{p+1}(x)/dx) \le 0$ because $\mu^p(x) > 0$, $\mu^{p+1}(x) > 0$, $d\mu^p(x)/dx \le 0$ and $d\mu^{p+1}(x)/dx \ge 0$. From the above results, we know that $(d\mu^p(x)/dx)\mu^{p+1}(x) - \mu^p(x)(d\mu^{p+1}(x)/dx) = 0$ except for the overlapped region.

Consequently, we establish that the set of PT membership functions built with complete and consistent fuzzy sets satisfies $(d\mu^p(x)/dx)\mu^q(x) \le \mu^p(x)(d\mu^q(x)/dx)$ for all $1 \le p < q \le M$ in the whole region. ■

PT membership functions using complete and consistent fuzzy sets include many commonly used membership functions such as triangular membership function, trapezoid membership function, bell-shaped membership function and so on. We can easily know that Seki's second condition is a special case of condition 3) of Theorem 1. But complete and consistent fuzzy set does not include one popular membership function such as Gaussian membership function. Lemma 2 suggests that Gaussian membership functions also satisfy condition 3) of Theorem 1 with some constraints.

Lemma 2: [6] The set of Gaussian membership functions

$$\mu^l(x : m^l, \sigma^l) = exp\left[-\frac{1}{2}\left(\frac{x - m^l}{\sigma^l}\right)^2\right]$$

where $1 \le l \le M$, $m^p \le m^q$ and $\sigma^p = \sigma^q$ for all $1 \le p < q \le M-1$ satisfies $(d\mu^p(x)/dx)\mu^q(x) \le \mu^p(x)(d\mu^q(x)/dx)$ for all $1 \le p < q \le M$.

Proof: For Gaussian membership functions, we know that

$$\frac{d\mu^l(x)}{dx} = -\frac{x - m^l}{(\sigma^l)^2}\mu^l(x).$$

Then we have

$$\frac{d\mu^p(x)}{dx}\mu^q(x) - \mu^p(x)\frac{d\mu^q(x)}{dx} = \frac{-x(\sigma^q)^2 + x(\sigma^p)^2 - (\sigma^p)^2 m^q + (\sigma^q)^2 m^p}{(\sigma^p)^2(\sigma^q)^2}\mu^p(x)\mu^q(x). \tag{4}$$

Equation (4) is nonpositive if $m^p \le m^q$ and $\sigma^p = \sigma^q$ for all $1 \le p < q \le M$. ■

IV. Simulation Example

In this section, we give an example to show that the derived conditions indeed make the functional-type SIRMs fuzzy system monotonic. We set IF-THEN rules for each SIRM as follows:

$$\text{SIRM1}: \begin{cases} \text{IF } x_1 = A_1^1 \text{ THEN } y_1 = 0.5x_1 - 1 \\ \text{IF } x_1 = A_1^2 \text{ THEN } y_1 = x_1 + 1 \\ \text{IF } x_1 = A_1^3 \text{ THEN } y_1 = 2x_1 + 3 \end{cases}$$

$$\text{SIRM2}: \begin{cases} \text{IF } x_2 = A_2^1 \text{ THEN } y_2 = x_2 + 1 \\ \text{IF } x_2 = A_2^2 \text{ THEN } y_2 = 2x_2 + 2 \\ \text{IF } x_2 = A_2^3 \text{ THEN } y_2 = 3x_2 + 3 \end{cases}$$

where $\mathbf{x} \in U = [0,1] \times [0,1]$. Then the consequent part parameters satisfy conditions 1) and 2) of Theorem 1. For this example, we use two types of membership functions: triangular membership functions and Gaussian membership functions.

A. Triangular membership function case

We assign three triangular membership functions for each input as follows:

$$\mu_1^1(x) = \mu_1^1(x; 0, 0, 0.5),$$
$$\mu_1^2(x) = \mu_1^2(x; 0.4, 0.7, 1),$$
$$\mu_1^3(x) = \mu_1^3(x; 0.8, 1, 1),$$

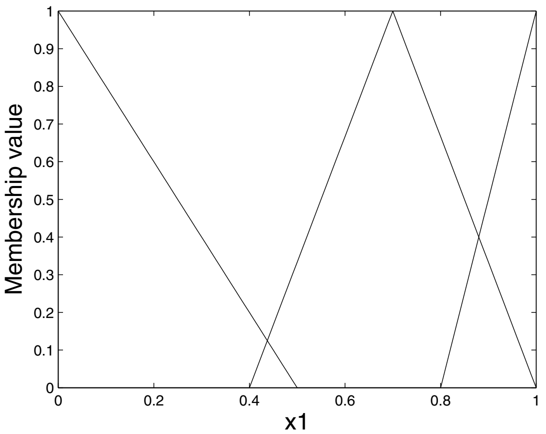

Fig. 5. Arrangement of triangular membership functions in x_1.

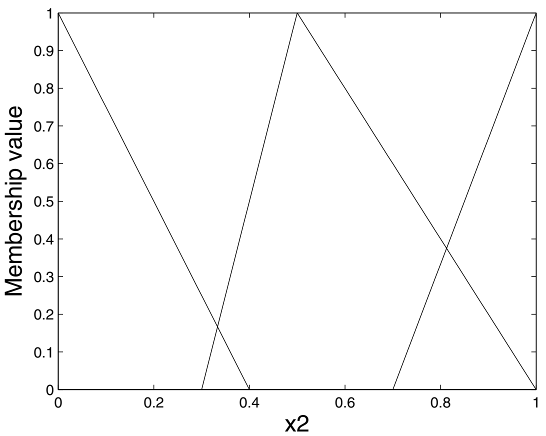

Fig. 6. Arrangement of triangular membership functions in x_2.

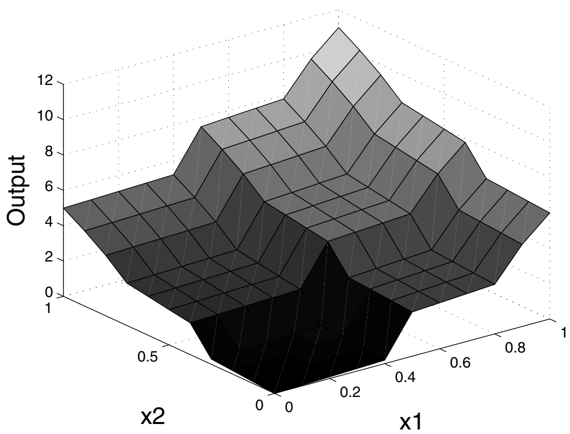

Fig. 7. Fuzzy system output when triangular membership functions are used.

and

$$
\begin{aligned}
\mu_2^1(x) &= \mu_2^1\left(x; 0, 0, 0.4\right), \\
\mu_2^2(x) &= \mu_2^2\left(x; 0.3, 0.5, 1\right), \\
\mu_2^3(x) &= \mu_2^3\left(x; 0.7, 1, 1\right).
\end{aligned}
$$

The arrangement of the triangular membership functions is complete and consistent as shown in Fig. 5 and Fig. 6. Therefore, this arrangement of input membership functions satisfies Lemma 1 or condition 3) of Theorem 1. Then, the output of the fuzzy system is monotonically increasing with respect to each input as shown in Fig. 7. Accordingly, the output of the fuzzy system is monotonically increasing with respect to its input.

B. Gaussian membership function case

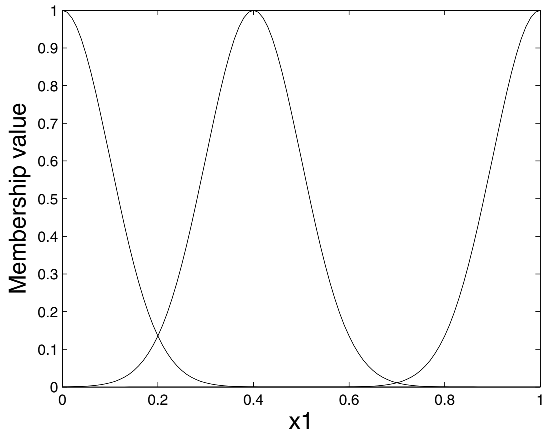

Fig. 8. Arrangement of Gaussian membership functions in x_1.

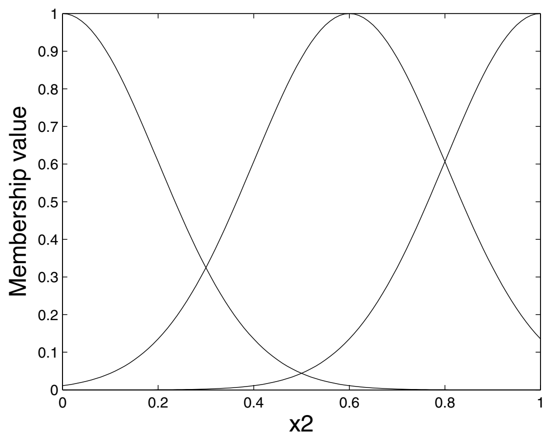

Fig. 9. Arrangement of Gaussian membership functions in x_2.

Next, we assign three Gaussian membership functions for each input axis as following:

$$
\begin{aligned}
\mu_1^1(x) &= \mu_1^1\left(x; [0.1, 0]\right), \\
\mu_1^2(x) &= \mu_1^2\left(x; [0.1, 0.4]\right), \\
\mu_1^3(x) &= \mu_1^3\left(x; [0.1, 1]\right),
\end{aligned}
$$

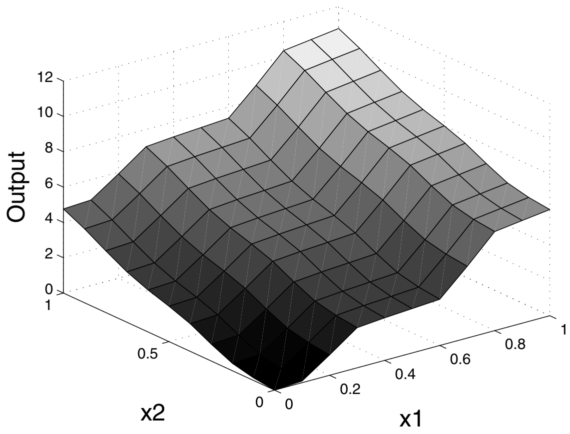

Fig. 10. Fuzzy system output when Gaussian membership functions are used.

and

$$\mu_2^1(x) = \mu_2^1(x;[0.2,0]),$$
$$\mu_2^2(x) = \mu_2^2(x;[0.2,0.6]),$$
$$\mu_2^3(x) = \mu_2^3(x;[0.2,1])$$

where those are depicted as Fig. 8 and Fig. 9 and each component in $[\cdots]$ is the standard deviation and the mean, respectively. Because the same standard deviation are used in each axis and the means are arranged by the increasing order, Lemma 2 is satisfied. The output of the fuzzy system is monotonically increasing with respect to each input as shown in Fig. 10. Accordingly, the output of the fuzzy system is monotonically increasing with respect to its input.

V. CONCLUSION

We studied the monotonicity of functional-type SIRMs connected fuzzy systems and derived sufficient conditions for the fuzzy systems to be monotonic. We, first, showed that the monotonic orderings in input fuzzy sets and consequent part cannot guarantee the monotonicity in the functional-type SIRMs connected fuzzy systems. The general conditions for the monotonicity are derived by differentiating the fuzzy output and setting the derivative to be nonnegative. The conditions are classified into two parts: one is on the consequent part parameters and the other is on the geometric constraints of the input membership functions. From the conditions, we could know that each consequent part of fuzzy rules must be monotonic to preserve the monotonicity and any popular membership functions can be used for the monotonicity. Simulation examples showed that the derived conditions establish the monotonicity to the functional-type SIRMs connected fuzzy systems.

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