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Adaptive learning control of uncertain robotic systems

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Adaptive learning control of uncertain robotic systems

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An adaptive learning control scheme is presented for uncertain robotic systems that is capable of tracking the entire profile of the reference input. The control scheme consists of three control blocks: a linear feedback, a feedforward error compensation and a learning strategy. At each iteration, the linear feedback with the feedforward error compensation provides stability of the system and keeps its state errors within uniform bounds. The learning strategy, on the other hand, estimates the desired control input and uncertain system parameters, which are used to track the entire span of a reference input over a sequence of iterations. In contrast with many other learning control techniques, the proposed learning algorithm neither uses derivative terms of feedback errors nor assumes any perturbations on the learning control input as a prerequisite. The parameter estimator neither uses any joint acceleration terms nor uses any inversion of the estimated inertia matrix, which makes its implementation practical. The proposed controller is superior to the high-gain feedback based learning controller (Kuc *et al.* 1991) because the magnitude of linear feedback gains required to guarantee convergence of the learning controller can be made much smaller thereby solving the problems of actuator saturation or actuator overdesign. The convergence proof of the learning scheme with or without parameter estimation is given under mild conditions on the feedback gains and learning control gains. Under the condition of persistent excitation in the domain of iteration sequence, it is proved that the estimated system parameters also converge to the true parameters.

1. Introduction

In recent years, many adaptive control techniques have been reported in the robotics literature as viable means to control uncertain systems for which the conventional PID-type control methods are not adequate. In particular, the parameter uncertainties such as link length, mass, inertia and frictional nonlinearity etc can be accommodated with a number of adaptive control means. However, most of these techniques have been developed under restrictive assumptions of one kind or another. Quite often the manipulator dynamics are assumed to be linear and/or decoupled at each joint. These assumptions will no longer be true if the manipulator is equipped with direct-drive actuators or performs high speed motion. In these cases, the nonlinear coupling terms and the gravity terms in the manipulator dynamics become significant. A number of adaptive control methods have been reported by Craig *et al.* (1986), Middleton and Goodwin (1988), Slotine and Lee (1989) and Spong and Ortega (1990) which accommodate nonlinear coupling and gravity terms in their control system design. Most of these control methods are based on the techniques which use the regression matrices to make the manipulator dynamics linear with respect to unknown parameters. Among these adaptive control techniques, Craig *et al.* (1986)

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assumed that the estimated inertial matrix is invertible and used joint accelerations in deriving their parameter estimators. Middleton and Goodwin (1988) used filtered signals in their adaptation loop and thereby eliminated the acceleration terms. Spong and Ortega (1990, 1991) proved that the invertibility assumption can be eliminated when acceleration signals are used. Slotine and Lee (1989) relaxed these restrictions and established the stable system by using the skew-symmetry property of $\dot{D}-2B$, where $\dot{D}$ is the time derivative of the inertia matrix and B is the velocity coupling matrix.

In parallel with the adaptive control techniques, there has been a substantial amount of research effort in the iterative learning control techniques. The idea behind the learning control is that the knowledge obtained from the previous trial is used to improve the control input for the next trial. Such a learning capability to improve system performance forms an essential control strategy of the human being.

Among the publications in the area of learning control, Craig *et al.* (1984) proposed a simple learning scheme which is based on the updated signal from a linear filter. Arimoto *et al.* (1984) developed a learning method in which the time derivative of system output error is used to modify the control input for the next iteration. Atkeson and McIntyre (1986) proposed a model-based learning scheme in which an estimated system is used to calculate the update signal. Miller III *et al.* (1987) applied the idea of CMAC memory (Albus 1981) to their learning rule and developed a general learning controller for robot manipulators. Their learning controller converges to an approximate inverse dynamic model of a robot manipulator around the input state trajectory. Bondi *et al.* (1988) developed a learning algorithm for robot systems which uses position, velocity and acceleration signals in updating the control input at each iteration. Their result is based on the high-gain feedback concept which sets up uniform upper bounds on the trajectory errors. Miyamoto *et al.* (1988) adopted a single layer perceptron concept in computing the feedforward torque components for a robot manipulator. In their scheme, subsystems are multiplied by a feedback error signal to approximate the uncertain parametric system.

Messner *et al.* (1990) proposed a feasible learning method which estimates the kernel of the inerse dynamics function of a robot system and generates a learning control input. In a recent work, Kuc *et al.* (1991) proposed a learning scheme which combines a conventional feedback controller and a feedforward learning controller. In this scheme, the learning controller is structurally simple, computationally fast, and does not use any acceleration terms. However, similar to that of Bondi *et al.* (1988), its convergence result is based on the stabilizing high-gain feedback controller, which is prone to actuator saturation and vulnerable to noise. Dawson *et al.* (1991) proposed a learning controller which consists of two parts: a computed torque servo for the modelled rigid body portion and a learning law for the unmodelled dynamics. They derived uniform bounds on the position errors and velocity errors but not the uniform convergence of these errors to zero. Qu *et al.* (1993) successfully developed a learning controller that guarantees uniform convergence of position, velocity and acceleration errors to zero. However, their method still relies on joint acceleration error terms in their leading controller. Saab (1994) proposed the so-called P-type learning controller for a nonlinear time-varying system. The main drawback in this paper is that the uniform convergence of output error is guaranteed only when the system trajectory satisfies some complicated conditions which is fairly difficult to implement. Ahn *et al.* (1993) and Jang *et al.* (1995) proposed iterative learning control laws based on the relative degree of nonlinear system introduced by Isidori (1989). In their learning

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controller, however, the learning gains are implicitly coupled with the decoupling matrix and the joint acceleration terms are used partly because the relative degree two is used in their robot control scheme.

In this paper, a new iterative learning control scheme is presented for mechanical robot manipulators which guarantees global stability of the system dynamics without acceleration measurement or estimation. The proposed scheme is an improvement over the high-gain feedback approach (Kuc *et al.* 1991, 1992) because the magnitude of the feedback gain can be made much smaller. The lower bound condition on the feedback gain simply implies that the parametric uncertainty embedded in the regression matrix should be taken into account by increasing the magnitude of feedback gain. In the learning controller, an estimator is incorporated in the domain of the iteration sequence to estimate unknown system parameters. The parameter estimator neither uses any system acceleration nor inversion of the estimated inertia matrix, so that the introduction of a filter or the invertibility assumption of the estimated inertia matrix are not required at all (Middleton and Goodwin 1988, Craig *et al.* 1986, Spong and Ortega 1990). Moreover, if the condition of persistent excitation (PE) is satisfied in the domain of iteration sequence, the convergence of system parameters is guaranteed, even when the PE condition in the time domain (Craig *et al.* 1986) is not satisfied. The time domain PE condition ceases to hold when the system trajectories are monotonic or when the duration (t_f) of the trajectories is small.

In the following, the following notations and definitions will be used. R^+ denotes the set of non-negative real numbers. R^n represents the n-dimensional vector space over R endowed with the euclidean or l_2 norm $|x| = (\sum_{i=1}^{n} x_i^2)^{1/2}$. For any $m \times n$ matrix B, the induced matrix norm $\|B\|$ is defined as $\|B\| = (\lambda_{\max}(B^T B))^{1/2}$, where $\lambda_{\max}(\cdot)$ is the largest eigenvalue. The truncated maximum norms for $|\cdot|$ and $\|\cdot\|$ are defined as $|\cdot|_m = \max_{0 \leqslant t \leqslant t_f} |\cdot|$ and $\|\cdot\|_m = \max_{0 \leqslant t \leqslant t_f} \|\cdot\|$, respectively. For symmetric matrices A and B, $A > (\geqslant) B$ implies that $\lambda_{\min}(A) > (\geqslant) \lambda_{\max}(B)$ where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the smallest and the largest eigenvalues, respectively. $C^2[0, t_f]$ is defined as the set of all real-valued vector functions in R^n whose second derivatives are piecewise continuous for all $t \in [0, t_f]$. $\equiv$ denotes 'is defined as'.

L_p denotes the set of Lebesgue measurable (or piecewise continuous) real valued vector functions whose elements are bounded in L_p norm, where

$$|\cdot|_p \equiv \begin{cases} \left(\int_0^\infty |\cdot|^p \, dt \right)^{1/p} & \text{for } p \in [1, \infty) \\ \sup_{t \geqslant 0} |\cdot| & \text{for } p = \infty \end{cases}$$

Similarly, $L_{pe}[0, t_f]$ is an extended subspace of L_p in which its elements are bounded in $L_{pe}[0, t_f]$ norm, where

$$|\cdot|_{pe} \equiv \begin{cases} \left(\int_0^{t_f} |\cdot|^p \, dt \right)^{1/p} & \text{for } p \in [1, \infty) \\ \sup_{0 \leqslant t \leqslant t_f} |\cdot| & \text{for } p = \infty. \end{cases}$$

Consider a dynamic system $\Sigma(U, X, Z)$, where $U\colon R^+ \to R^m$ represents the set of admissible inputs, $X \subset R^n$ and $Z \subset R^m$ are the set of states and the set of output values, respectively. The system Σ is said to be *passive* with respect to the pair $\{u, z\}$ if there exists a non-negative scalar function $V(x) \in C[0, t_f]$ with $V(0) = 0$ such that (Byrnes *et al.* 1991, Desor and Vidyasagar 1975)

$$V(x) - V(x_0) \leqslant \int_0^t u^T z \, d\tau$$

where $x, x_0 \in X$ and $x = \phi(x_0, u, t)$. Note that if $u = 0$, V is decreasing with t increasing. Note also that if the system is passive and its zero dynamics are exponentially stable, then the negative output feedback $u = -z$ stabilizes the overall system.

The system Σ is said to be *strictly passive* if it is passive and there exists a positive definite scalar function W such that

$$V(x) - V(x_0) = \int_0^t u^T z \, d\tau - \int_0^t W(x) \, d\tau$$

The dynamic system Σ with the input/output pair $\{u/z\}$ is said to be input/output $L_p(L_{pe}[0, t_f])$ stable if and only if there exist constants $\beta_1 > 0$, $\beta_2 \geqslant 0$ such that

$$|z|_p(|z|_{pe}) \leqslant \beta_1 |u|_p(|u|_{pe}) + \beta_2$$

The dynamic system Σ with the input/output pair $\{u/z\}$ is said to be positive real if the pair $\{u/z\}$ satisfies the inequality

$$\int_0^t u^T z \, d\tau \geqslant 0 \quad \text{for all } t \in [0, t_f]$$

where $x(0) = 0$. Note that a positive real system Σ is passive if the system is causal or a state of the system is reachable from zero.

A matrix function Y^j: $R^+ \to R^{n \times l}$ is *persistently exciting in the domain of iteration sequence* if and only if there exists positive constants α_1, α_2 and a positive integer N such that (Kuc and Lee 1991)

$$\alpha_1 I \leqslant \sum_{i=j}^{j+N} Y^{i^T}(t) \, Y^i(t) \leqslant \alpha_2 I$$

for $t \in [0, t_f]$.

2. Problem formulation

Consider a robot system with n rigid bodies, the mathematical model of which is

$$D(q(t)) \ddot{q}(t) + B(q(t), \dot{q}(t)) \dot{q}(t) + F(q(t), \dot{q}(t)) + T_a(t) = T(t) \tag{1}$$

where $q(t) \in R^n$ is the generalized joint coordinate vector, $D(q(t)) \in R^{n \times n}$ is the inertia matrix, $B(q(t), \dot{q}(t)) \dot{q}(t) \in R^n$ is the centripetal plus Coriolis force vector,

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$F(q(t), \dot{q}(t)) \in R^n$ is the gravitational plus frictional forces, $T(t) \in R^n$ is the joint control input vector, and $T_a(t) \in R^n$ is the unknown disturbance vector which is assumed to be bounded. The symmetric inertia matrix $D(q(t)) \in R^{n \times n}$ is assumed to be positive definite and bounded as

$$0 < \lambda_1 I \leqslant D(q(t)) \leqslant \lambda_2 I \quad \text{for all } t \in [0, t_f]$$

where $\lambda_1, \lambda_2 > 0$ and I is an $n \times n$ identity matrix. The matrix $\dot{D}(q(t)) - 2B(q(t), \dot{q}(t))$ is assumed to be skew-symmetric, which implies

$$z^T(\dot{D} - 2B)z = 0 \quad \text{for all non-zero } z \in R^n$$

Now, when the desired trajectory $q_d(t) \in R^n$ is specified as a reference input for system (1), the fundamental control problem is to find a control input $T(t)$ with which the system output $q(t)$ follows $q_d(t)$ for all $t \in [0, t_f]$ as close as possible. In the framework of learning control, this objective can be stated as follows.

Problem statement: Suppose that $q_d(t) \in C^2[0, t_f]$, the trajectory vector of system (1), is in the interior of a domain Q, which is a closed, bounded and simply connected subset of R^n. Then, find a sequence of piecewise continuous control command vector $T^j(t) \in R^n (t \in [0, t_f])$ for uncertain system (1) with which the system trajectory $q^j(t)$ follows $q_d(t)$ with a given accuracy ε as follows

$$|q_d(t) - q^j(t)| \leqslant \varepsilon \quad \text{for all } t \in [0, t_f]$$

where j denotes the jth iteration.

In the following, the uncertain system (1) is assumed to be repetitive for all $t \in [0, t_f]$ and the operating conditions such as sampling frequency, payload scheduling, initial configuration etc, are all assumed to be prespecified. The desired joint position, velocity, acceleration and control input vectors are denoted as $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ and $T_d(t)$, respectively and the actual joint position, velocity, acceleration and control input vectors at the jth iteration are denoted as $q^j(t)$, $\dot{q}^j(t)$, $\ddot{q}^j(t)$ and $T^j(t)$, respectively. For notational brevity, the time argument t will be omitted in the following.

3. Feedforward learning control

The tracking capability of the iterative learning process hinges upon the stability of the closed-loop system at each iteration. Under certain conditions, the stability of the closed-loop system can be achieved with high-gain feedback (Kuc *et al.* 1992). However, due to physical constraints such as actuator saturation, the high-gain feedback scheme is difficult to use in actual physical systems. To overcome the problem, the proposed learning control scheme uses a feedback plus feedforward controller which, when applied to system (1), forms a stable closed-loop system and limits the tracking error within an acceptable bound at each iteration.

In our learning control scheme, the control setpoint vector T^j for the jth iteration is a combination of three input components

$$T^j = E^j + C^j + H^j \tag{2}$$

where E^j is the feedback control input, C^j is the computed torque error, and H^j is the learning control input. As stated in the problem statement, T^j is the joint control

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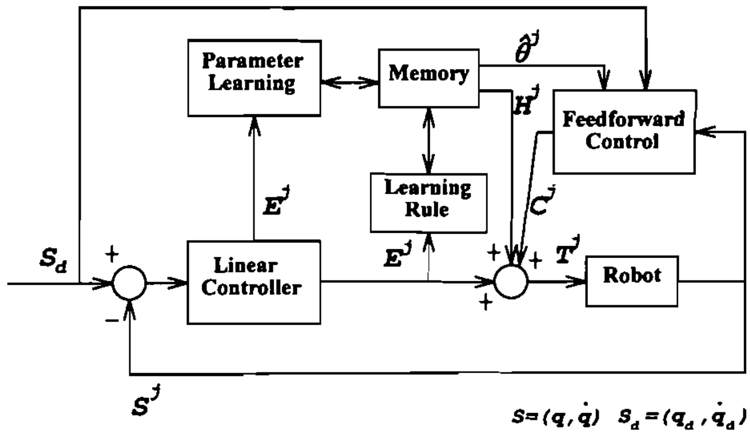

Schematic of the learning control systems.

torque vector that is to be applied to the joint actuator of the robot manipulator. The Figure shows the closed-loop dynamic system which implements this control strategy. The feedback control input E^j is responsible for the stabilization of the closed-loop system and is computed from the conventional proportional plus derivative (PD) control algorithm

$$E^j = L(\dot{e}^j + ae^j) \tag{3}$$

where $e^j \equiv q_d - q^j$, L is a symmetric positive definite matrix and a is a positive scale factor. The feedforward control input C^j compensates for the nonlinear part of the uncertain robotic system and helps keep the feedback gain of E^j reasonably small.

When the system parameters of (1) are completely known, the feedforward control input C^j is computed as follows:

$$C^j = D_e(q^j)\ddot{q}_d + B_e(q^j,\dot{q}^j)\dot{q}_d + F_e(q^j,\dot{q}^j) + a(D(q^j)\dot{e}^j + B(q^j,\dot{q}^j)e^j) \tag{4}$$

where $D_e(q^j) \equiv D(q^j) - D(q_d)$, $B_e(q^j,\dot{q}^j) \equiv B(q^j,\dot{q}^j) - B(q_d,\dot{q}_d)$ and $F_e(q^j,\dot{q}^j) \equiv F(q^j,\dot{q}^j) - F(q_d,\dot{q}_d)$. Note that the C^j term generates the computed torque errors not the desired torques as many feedforward control methods do.

Now, substituting the torque input (2) into (1), we have

$$D(q^j)\ddot{q}^j + B(q^j,\dot{q}^j)\dot{q}^j + F(q^j,\dot{q}^j) + T_a = E^j + C^j + H^j \tag{5}$$

Applying (3) and (4) into (5), we obtain

$$\begin{aligned} &D(q^j)(\ddot{q}_d - \ddot{e}^j) + B(q^j,\dot{q}^j)(\dot{q}_d - \dot{e}^j) + F(q^j,\dot{q}^j) + T_a \\ &\quad = D_e(q^j)\ddot{q}_d + B_e(q^j,\dot{q}^j)\dot{q}_d + F_e(q^j,\dot{q}^j) + a(D(q^j)\dot{e}^j + B(q^j,\dot{q}^j)e^j) + L(\dot{e}^j + ae^j) + H^j \end{aligned}$$

If we define z^j as $\dot{e}^j + ae^j$ with a positive constant a, then the above equation becomes

$$\begin{aligned} D(q^j)\dot{z}^j + B(q^j,\dot{q}^j)z^j + Lz^j &= D(q_d)\ddot{q}_d + B(q_d,\dot{q}_d)\dot{q}_d + F(q_d,\dot{q}_d) + T_a - H^j \\ &= T_d^* + T_a - H^j \\ &= T_d - H^j \\ &= \tilde{U}^j \end{aligned} \tag{6}$$

where $T_d^* \equiv D(q_d)\ddot{q}_d + B(q_d,\dot{q}_d)\dot{q}_d + F(q_d,\dot{q}_d)$, $T_d \equiv T_d^* + T_a$ and $\tilde{U}^j \equiv T_d - H^j$. Here, T_d^* is the desired control input without any external disturbance while T_d is the same

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desired control input with external disturbance T_a. The main function of H^j is in the estimation of T_d and its compensation. The question is then how to update the learning control input H^j to compensate for T_d. As a motivation to generate a learning algorithm, let us define a Lyapunov function candidate $W(z^j)$ as $\frac{1}{2}z^{j^{\mathrm{T}}}D(q^j)\,z^j$. Then, the derivative of $W(z^j)$ along the error trajectory is

$$\begin{aligned}\dot{W}(z^j) &= z^{j^{\mathrm{T}}}D(q^j)\,\dot{z}^j+\tfrac{1}{2}z^{j^{\mathrm{T}}}\dot{D}(q^j)\,z^j\\ &= z^{j^{\mathrm{T}}}(\tilde{U}^j-B(q^j,\dot{q}^j)\,z^j-Lz^j)+\tfrac{1}{2}z^{j^{\mathrm{T}}}\dot{D}(q^j)\,z^j \quad \text{(from (6))}\\ &= \tfrac{1}{2}z^{j^{\mathrm{T}}}(\dot{D}(q^j)-2B(q^j,\dot{q}^j))\,z^j-z^{j^{\mathrm{T}}}Lz^j+z^{j^{\mathrm{T}}}\tilde{U}^j\\ &= -z^{j^{\mathrm{T}}}Lz^j+z^{j^{\mathrm{T}}}\tilde{U}^j\end{aligned}$$

where the skew-symmetry property of $\dot{D}(q^j)-2B(q^j,\dot{q}^j)$ has been used.

Integrating both sides of the above equation, we obtain

$$W(z^j(t))-W(z^j(0)) = -\int_0^t z^{j^{\mathrm{T}}}Lz^j\,\mathrm{d}\tau+\int_0^t z^{j^{\mathrm{T}}}\tilde{U}^j\,\mathrm{d}\tau$$

which indicates that the error dynamics are strictly passive with respect to the pair $\{\tilde{U}^j/z^j\}$. Hence, if we use $-z^j$ for $\tilde{U}^j$, then the error dynamics converge to zero. However, this is not physically realizable because the disturbance vector T_a is unknown.

3.1. *Feedforward predictive learning*

In order to derive one physically realizable control law, we propose the following control algorithm in recursive form

$$\tilde{U}^{j+1} = \tilde{U}^j-\beta Lz^j \text{ or } H^{j+1} = H^j+\beta E^j \quad (j \geqslant 1) \tag{7}$$

where β is a training factor, which is set to $0<\beta<2$. As an initial condition, $z^j(0)$ is set to 0 (i.e. $e^j(0)=0$ and $\dot{e}^j(0)=0$) for all $j=1,2,\ldots$ and H^1 is set to T_d^* which can be computed *a priori*. The constructed update rule is what we call the prediction learning rule, since the learning input H^{j+1} at the $(j+1)$th iteration is predicted from the information available at the jth iteration. The prediction learning rule has been introduced in its preliminary form in Kuc and Lee (1991) but is revised here in its refined form with complete proofs.

Now, in order for the proposed learning scheme to be meaningful, the error dynamics of the closed-loop system must be in the stable state at every iteration. As the following lemma shows, the tracking error z^j can be made bounded whenever $\tilde{U}^j$ is bounded.

Lemma 1: *Assume that* $\tilde{U}^j \in L_{pe}[0,t_f]$. *Then, the error dynamics* (6) *with the prediction learning rule* (7) *is* L_{pe} *stable with respect to the pair* $\{\tilde{U}^j/z^j\}$.

Proof: For the proof, see Appendix A. □

According to Lemma 1, the tracking error of the closed-loop system is bounded at the first iteration whenever the unknown disturbance $T_a(=\tilde{U}^1)$ is bounded. Therefore, as the iteration continues, the learning controller converges, as is made formal in the following theorem.

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Theorem 1: *Assume that the disturbance vector* $T_a \in L_{2e}[0, t_f] \cap L_{\infty e}[0, t_f]$. *Then the learning control algorithm constructed above for the robot dynamic system* (1) *with known parameters converges uniformly as follows:*

$$\text{(i)}\quad V^{j+1}(t) \leqslant V^j(t)$$

$$\text{(ii)}\quad \lim_{j\to\infty} q^j(t) = q_d(t)$$

$$\text{(iii)}\quad \lim_{j\to\infty} \dot{q}^j(t) = \dot{q}_d(t) \quad \text{for all } t \in [0, t_f]$$

where V^j *is the performance index functional*

$$V^j(t) \equiv \int_0^t \tilde{U}^{j\mathrm{T}}(\tau)\, L^{-1} \tilde{U}^j(\tau)\, d\tau$$

Proof: For the proof, see Appendix B. □

Note here that V^j is the weighted extended L_2-norm of difference of desired input T_d minus learning input H^j. The feasibility of the learning system can also be stated in terms of the passivity property of the learning system.

Corollary 1: *The overall learning system* (6) *with the prediction learning rule is passive with respect to the pair* $\{\tilde{U}^j/z^j\}$.

Proof: The proof of Theorem 1 states that

$$\Delta V^j = \int_0^t (\beta^2 z^{j\mathrm{T}} L z^j - 2\beta z^{j\mathrm{T}} \tilde{U}^j)\, d\tau \leqslant 0$$

which is

$$2\beta \int_0^t \tilde{U}^{j\mathrm{T}} z^j\, d\tau \geqslant \beta^2 \int_0^t z^{j\mathrm{T}} L z^j\, d\tau \geqslant 0$$

where $z^j(0) = 0$. □

In the proof of Theorem 1, it is stated that the error signals are bounded at each iteration. This fact is made more precise in the following corollary.

Corollary 2: *In the feedforward learning control systems with the prediction learning rule,* $z^j(t)$ *and* $\tilde{U}^j \in L_{2e}[0, t_f] \cap L_{\infty e}[0, t_f]$ *and* $\dot{z}^j(t) \in L_{\infty e}[0, t_f]$. *Moreover, if* $\dot{T}_d^*$ *and* $\dot{T}_a \in L_{\infty e}[0, t_f]$, *then* $\dot{\tilde{U}}^j \in L_{\infty e}[0, t_f]$.

Proof: For the proof, see Appendix C. □

3.2. *Feedforward current learning*

When E^{j+1} is used instead of E^j in the learning rule (7), we obtain the current learning rule

$$H^{j+1} = H^j + \beta E^{j+1} \tag{8}$$

where $\beta > 0$ and $z^j(0) = 0$ (i.e. $e^j(0) = 0$ and $\dot{e}^j(0) = 0$) for all $j \geqslant 1$. As in the prediction learning rule, H^1 is set to T_d^* for all $t \in [0, t_f]$.

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The current learning rule has an advantage over the prediction learning rule because the learning gain β can be set to any positive value. The operation characteristics of the feedforward learning control with the current learning rule are summarized in the following theorem.

Theorem 2: *Assume that T_a is bounded as in Theorem 1. When the system parameters are known, the current learning controller, which is composed of the error terms* (3), (4) *and the learning rule* (8), *converges uniformly as follows:*

$$\text{(i)}\quad V^{j+1}(t) \leqslant V^j(t)$$

$$\text{(ii)}\quad \lim_{j\to\infty} q^j(t) = q_d(t)$$

$$\text{(iii)}\quad \lim_{j\to\infty} \dot{q}^j(t) = \dot{q}_d(t) \quad \text{for all } t\in[0, t_f]$$

where V^j is defined in Theorem 1.

Proof: For the proof, see Appendix D.

4. Adaptive learning control

When the system parameters of (1) are not completely known, a parameter learning rule is required to estimate the unknown system parameters in the system. In order to derive a parameter learning rule for the adaptive learning system, we make use of the linear parameterization technique for the dynamic system (1). That is, rearranging the dynamic equation (1) in terms of a set of system parameters, we obtain an algebraic description of the system

$$Y(q(t), \dot{q}(t), \ddot{q}(t))\,\theta = T(t) - T_a(t)$$

where $Y(q(t), \dot{q}(t), \ddot{q}(t)) \in R^{n\times l}$ is the regression matrix and $\theta \in R^l$ is a suitably chosen parameter vector.

Because the system parameters θ are not known, the feedforward error input term C^j depends on the estimated parameter vector $\hat{\theta}^j$

$$C^j = \hat{D}_e(q^j)\,\ddot{q}_d + \hat{B}_e(q^j, \dot{q}^j)\,\dot{q}_d + \hat{F}_e(q^j, \dot{q}^j) + a(\hat{D}(q^j)\,\dot{e}^j + \hat{B}(q^j, \dot{q}^j)\,e^j) \tag{9}$$

where $\hat{D}_e(q^j) \equiv \hat{D}(q^j) - \hat{D}(q_d)$, $\hat{B}_e(q^j, \dot{q}^j) \equiv \hat{B}(q^j, \dot{q}^j) - \hat{B}(q_d, \dot{q}_d)$ and $\hat{F}_e(q^j, \dot{q}^j) \equiv \hat{F}(q^j, \dot{q}^j) - \hat{F}(q_d, \dot{q}_d)$, respectively. Note here that $\hat{}$ denotes an estimated system. Substituting (3) and (9) into (5) and using the definition $z^j \equiv \dot{e}^j + ae^j$, we obtain

$$\begin{aligned} D(q^j)\,\dot{z}^j + B(q^j, \dot{q}^j)\,z^j + Lz^j = {} & D(q^j)\,\ddot{q}_d + B(q^j, \dot{q}^j)\,\dot{q}_d + F(q^j, \dot{q}^j) + T_a \\ & - \hat{D}_e(q^j)\,\ddot{q}_d - \hat{B}_e(q^j, \dot{q}^j)\,\dot{q}_d - \hat{F}_e(q^j, \dot{q}^j) + a(\tilde{D}(q^j)\,\dot{e}^j + \tilde{B}(q^j, \dot{q}^j)\,e^j) - H^j \end{aligned}$$

where $\tilde{D}(q^j) = D(q^j) - \hat{D}(q^j)$ and $\tilde{B}(q^j, \dot{q}^j) = B(q^j, \dot{q}^j) - \hat{B}(q^j, \dot{q}^j)$. Simplifying the above equation, we have

$$\begin{aligned} D(q^j)\,\dot{z}^j + B(q^j, \dot{q}^j)\,z^j + Lz^j = {} & D_e(q^j)\,\ddot{q}_d + B_e(q^j, \dot{q}^j)\,\dot{q}_d + F_e(q^j, \dot{q}^j) + T_d \\ & - \hat{D}_e(q^j)\,\ddot{q}_d - \hat{B}_e(q^j, \dot{q}^j)\,\dot{q}_d - \hat{F}_e(q^j, \dot{q}^j) + a(\tilde{D}(q^j)\,\dot{e}^j + \tilde{B}(q^j, \dot{q}^j)\,e^j) - H^j \\ = {} & \tilde{D}_e(q^j)\,\ddot{q}_d + \hat{B}_e(q^j, \dot{q}^j)\,\dot{q}_d + \tilde{F}_e(q^j, \dot{q}^j) + a(\tilde{D}(q^j)\,\dot{e}^j + \tilde{B}(q^j, \dot{q}^j)\,e^j) + \tilde{U}^j \end{aligned} \tag{10}$$

where $\tilde{D}_e(q^j) = D_e(q^j) - \hat{D}_e(q^j)$, $\tilde{B}_e(q^j, \dot{q}^j) = B_e(q^j, \dot{q}^j) - \hat{B}_e(q^j, \dot{q}^j)$, $\tilde{F}_e(q^j, \dot{q}^j) = F_e(q^j, \dot{q}^j) - \hat{F}_e(q^j, \dot{q}^j)$ and $\tilde{U}^j = T_d - H^j$, respectively.

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Rearranging the right-hand side of (10) in terms of suitably chosen system parameters, we obtain

$$D(q^j)\dot{z}^j + B(q^j, \dot{q}^j) z^j + Lz^j = Y_e^j \tilde{\theta}^j + \tilde{U}^j \tag{11}$$

where $Y_e^j = Y(q^j, \dot{q}^j, q_d, \dot{q}_d, \ddot{q}_d) \in R^{n \times l}$ is the regression matrix and $\tilde{\theta}^j = \theta - \hat{\theta}^j$ is a parameter error vector. From the construction, the regression matrix Y_e^j converges to a null matrix, when the system states $\{q^j, \dot{q}^j\}$ converge to $\{q_d, \dot{q}_d\}$.

If the right-hand side of (11) is zero, the system would be asymptotically stable (Slotine and Lee 1989, Spong and Vidyasagar 1989). However, the presence of the bias terms keeps the tracking error z^j from being zero and the magnitude of error depends on feedback gains as well as bias terms. Hence, our strategy in the tracking problem of uncertain systems is in constructing a learning mechanism to estimate and cancel the bias terms. That is, we need a learning rule for the estimation of the unknown system parameters θ as well as for the computation of the learning input H^j. In order to construct a learning rule, the strict passivity property of uncertain error dynamics (equation (11)) is used again. That is, if we define $W(z^j)$ as $\frac{1}{2}z^{j\mathrm{T}} D(q^j) z^j$ as in the previous section, then the pair $\{(\tilde{U}^j + Y_e^j \tilde{\theta}^j)/z^j\}$ satisfies

$$W(z^j(t)) - W(z^j(0)) = -\int_0^t z^{j\mathrm{T}} L z^j \, \mathrm{d}\tau + \int_0^t z^{j\mathrm{T}} (\tilde{U}^j + Y_e^j \tilde{\theta}^j) \, \mathrm{d}\tau$$

If we set $\tilde{U}^j$ to $-z^j$ (i.e. $H^j = T_d + z^j$) and $\tilde{\theta}^j$ to $-Y_e^{j\mathrm{T}} z^j$ (i.e. $\hat{\theta}^j = \theta + Y_e^{j\mathrm{T}} z^j$) in the above equation, the uncertain error dynamics (11) converges. However, they are not physically realizable because the disturbance T_a and the system parameter vector θ are not known.

4.1. *Adaptive predictive learning*

As in the previous case, we propose physically realizable learning rules for input update and parameter estimation as $\tilde{U}^{j+1} = \tilde{U}^j - \beta L z^j$ and $\tilde{\theta}^{j+1} = \tilde{\theta}^j - \beta S^{-1} Y_e^{j\mathrm{T}} z^j$ for $j \geqslant 1$, where S represents a symmetric positive definite gain matrix. The parameter estimation rule for the unknown system parameter vector θ is referred to as the prediction parameter estimator

$$\hat{\theta}^{j+1} = \hat{\theta}^j + \beta S^{-1} Y_e^{j\mathrm{T}} z^j \quad \text{for } j \geqslant 1 \tag{12}$$

and

$$\hat{\theta}^1 = \int_0^t (\beta (Y_e^1 + Y_d)^{\mathrm{T}} z^1 - \sigma_s \hat{\theta}^1) \, \mathrm{d}\tau \quad \text{for all } t \in [0, t_f]$$

where

$$\sigma_s = \begin{cases} 0 & \text{if } |\hat{\theta}^1| < \theta_0 \\ \dfrac{|\hat{\theta}^1|}{\theta_0} - 1 & \text{if } \theta_0 \leqslant |\hat{\theta}^1| \leqslant 2\theta_0 \\ 1 & \text{if } |\hat{\theta}^1| > 2\theta_0 \end{cases}, \quad \theta_0 > |\theta|, \quad \text{and} \quad Y_d \tilde{\theta}^1 = T_d^* - \hat{T}_d^{*1}$$

The vector $\hat{T}_d^{*1}$ represents the estimate of T_d^* at $j = 1$. The algorithm at the first iteration is the estimator originally proposed by Reed and Ioannou (1989), which

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guarantees that $\hat{\theta}^1$ is bounded even when the disturbance T_a exists. The gain matrix S can be used in scaling the lower bound of feedback gain L which is needed to guarantee the convergence of the learning process. For example, S can be chosen as

$$S = \alpha I + Y_e^{1^T} Y_e^1$$

for a positive constant α. Notice that, in the learning rule (12), the measurement of the acceleration as well as the inversion of estimated inertia matrix are not used at all (Craig *et al.* 1986).

When the proposed prediction parameter estimator and the prediction learning rule are used for the error system (11), the boundedness of the parameter error as well as the output error of the system, at the first iteration, depend on the disturbance and feedback gain L as shown below.

Lemma 2: *Assume that* $T_a \in L_{\infty e}$. *When the initial learning input* H^1 *is* $\hat{T}_d^{*1}$, *the error system* (11) *with the parameter estimator and the prediction learning rule is BIBO (bounded input bounded output) stable with respect to the pair* $\{T_a/z^j\}$ *at* $j = 1$.

Proof: For the proof, see Appendix E. □

Theorem 3: *Assume that* T_a *is bounded as in Theorem 1 and the initial input for the prediction learning rule is given by* $H^1 = \hat{T}_d^{*1}$. *Let the adaptive learning controller be composed of the feedback controller* (3), *the feedforward controller* (9) *and the learning controller* (7) *and the parameter estimator* (12). *If the symmetric positive definite matrix* L *satisfies*

$$D_0^j \equiv ((2-\beta)L - Y_e^j S^{-1} Y_e^{j^T}) > 0 \quad \textit{for all } t \in [0, t_f] \tag{13}$$

then the adaptive learning controller for the uncertain dynamic system (1) *converges uniformly as follows*:

$$\text{(i)} \quad V_a^{j+1}(t) \leqslant V_a^j(t)$$

$$\text{(ii)} \quad \lim_{j\to\infty} q^j(t) = q_d(t)$$

$$\text{(iii)} \quad \lim_{j\to\infty} \dot{q}^j(t) = \dot{q}_d(t) \quad \textit{for all } t \in [0, t_f]$$

where

$$V_a^j(t) \equiv \int_0^t (\tilde{U}^{j^T}(\tau) L^{-1} \tilde{U}^j(\tau) + \tilde{\theta}^{j^T}(\tau) S \tilde{\theta}^j(\tau))\, d\tau$$

for all $t \in [0, t_f]$ *and for all* $j \geqslant 1$.

Proof: For the proof, see Appendix F. □

Note in (13) that the feedback gain L must be set larger than $(1/(2-\beta))\, Y_e^j S^{-1} Y_e^{j^T}$, which is due to the parametric uncertainty embedded in the regression matrix Y_e^j. As in the feedforward learning case, the feasibility of the adaptive learning control algorithm can be verified in terms of the passivity property of the learning controller as follows.

Corollary 3: *Using the prediction learning rule and prediction parameter estimator, the overall adaptive learning system is positive real* (*or passive*) *with respect to the pair* $\{(\tilde{U}^j + Y_e^j \tilde{\theta}^j)/z^j\}$.

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The proof is similar to Corollary 1 and is obtained from Theorem 3. On the other hand, as in the feedforward learning case, the error signals are bounded at each iteration as follows.

Corollary 4: *In the adaptive learning controller with prediction learning rule and prediction parameter estimator,* $z^j(t)$, $\tilde{\theta}^j$ *and* $\tilde{U}^j \in L_{2e}[0, t_f] \cap L_{\infty e}[0, t_f]$ *and* $\dot{z}^j(t) \in L_{\infty e}[0, t_f]$. *If* $\dot{\theta}$, $\dot{T}_d^*$ *and* $\dot{T}_a \in L_{\infty e}[0, t_f]$, *then* $\dot{\hat{\theta}}^j$ *and* $\dot{\tilde{U}}^j \in L_{\infty e}[0, t_f]$.

The proof is similar to Corollary 2 except for the fact that the error equation (6) is replaced by (11) and the prediction parameter estimator is used in addition to the prediction learning rule. Compared with the feedforward learning case, the desired input T_d^* in Theorem 3 is not known *a priori* due to parametric uncertainty. Therefore, the main role of prediction learning in the adaptive learning case is in estimating the whole profile of T_d^* as well as the disturbance vector T_a.

4.2. *Adaptive current learning*

When the current learning rule (8) is used instead of the prediction learning rule (7), the error dynamics equation becomes

$$D(q^{j+1})\dot{z}^{j+1} + B(q^{j+1}, \dot{q}^{j+1})z^{j+1} + (1+\beta)Lz^{j+1} = \tilde{U}^j + Y_e^{j+1}\tilde{\theta}^{j+1} \tag{14}$$

For the current learning control system, the unknown parameters are to be estimated using the current parameter estimator

$$\hat{\theta}^{j+1} = \hat{\theta}^j + \beta S^{-1} Y_e^{j+1^{\mathrm{T}}} z^{j+1} \quad \text{for } j \geqslant 1 \tag{15}$$

and

$$\hat{\theta}^1 = \int_0^t (\beta(Y_e^1 + Y_d)^{\mathrm{T}} z^1 - \sigma_s \hat{\theta}^1)\,\mathrm{d}\tau \quad \text{for all } t \in [0, t_f]$$

where $Y_e^{j+1} \equiv Y(q^{j+1}, \dot{q}^{j+1}, q_d, \dot{q}_d, \ddot{q}_d)$ and σ_s is the same as in the prediction learning estimator. When the current learning rules (8) and (15) are used, the following facts can be stated for the adaptive current learning system.

Theorem 4: *When* T_a *is bounded as in Theorem 1, the learning controller which is generated from equations* (3), (8), (9) *and* (15) *converges uniformly as follows:*

$$\text{(i)}\quad V_a^{j+1}(t) \leqslant V_a^j(t)$$

$$\text{(ii)}\quad \lim_{j\to\infty} q^j(t) = q_d(t)$$

$$\text{(iii)}\quad \lim_{j\to\infty} \dot{q}^j(t) = \dot{q}_d(t) \quad \textit{for all } t \in [0, t_f]$$

where $V_a^j(t)$ *is defined in Theorem* 3.

Proof: For the proof, see Appendix G. □

Note that the lower bound constraint imposed on the feedback gain L has been removed in this result. This is due to the fact that the current error $z^{j+1}(t)$ is used in the current learning rule and the more recent information is used in the current parameter estimator.

The operating characteristics of the adaptive current learning system such as passivity and time-domain boundedness of the error signals can be established following the same arguments as in the adaptive prediction learning case.

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5. Convergence of parameter estimators

In the previous section, the output error of the adaptive learning system converges to zero without explicit parameter convergence of the parameter estimators. The controller just guarantees the boundedness of the estimated system parameters. In order to establish the convergence of the estimated parameters with the prediction parameter estimator or with the current parameter estimator, the persistent excitation (PE) condition of the system trajectory in the domain of iteration sequence is required as shown below.

Theorem 5: *Let* $Y_e^j: R^+ \to R^{n \times l}$ *be persistently exciting in the domain of iteration sequence. Then, the parameter estimator* (12) *or* (15) *converges for all* $t \in [0, t_f]$. *That is*

$$\lim_{j \to \infty} \hat{\theta}^j = \theta \quad \text{for all } t \in [0, t_f]$$

Proof: For the proof, see Appendix H. □

In view of Theorem 5, it follows that

$$\lim_{j \to \infty} \sum_{i=1}^{j} Y_e^{i^T} z^i = \frac{1}{\beta} S(\theta - \hat{\theta}^1) \quad \text{for all } t \in [0, t_f]$$

This means that the convergence of the parameter estimator holds for the entire time span of the system trajectory, which is in contrast to the time domain PE condition (Craig *et al.* 1986).

6. Concluding remarks

A new learning control scheme for a class of uncertain robotic systems is presented in this paper. The attractive features such as precise tracking and feedforward learning of inverse dynamics are achieved in the proposed controller under minor conditions in the feedback control gain. In particular, the parameter estimator of the proposed learning control scheme does not use any system acceleration or inversion of the estimated inertia matrix. The proposed controller is superior to the high-gain feedback based learning controller (Kuc *et al.* 1991) because the magnitude of feedback gains required to guarantee the convergence of the learning controller can be made much smaller, thereby solving the problems of actuator saturation or actuator overdesign. The price to pay for this advantage is the complex controller structure of the proposed controller, which normally required much more computation power and memory for the generation of real-time control input. The price for the CPU power and memory, however, is getting cheaper as faster processors become available at lower prices. At the same time, certain computationally-efficient algorithms may also be adopted for implementation of the adaptive learning control structure. Finally, an example of computer simulation with the adaptive learning control structure can be found in Kuc *et al.* (1991) in which external disturbances are shown to be accommodated by the proposed learning control scheme.

Appendix A: Proof of Lemma 1

If we set a Lyapunov function candidate to $W^j = W(z^j) = \frac{1}{2} z^{j^T} D z^j$, then along the error trajectory

$$\begin{aligned}\dot{W}^j &= \tfrac{1}{2}z^{j^{\mathrm{T}}}\dot{D}z^j + z^{j^{\mathrm{T}}}D\dot{z}^j \\ &= \tfrac{1}{2}z^{j^{\mathrm{T}}}\dot{D}z^j + z^{j^{\mathrm{T}}}(\tilde{U}^j - Bz^j - Lz^j) \\ &= \tfrac{1}{2}z^{j^{\mathrm{T}}}(\dot{D}-2B)\,z^j + z^{j^{\mathrm{T}}}(\tilde{U}^j - Lz^j) \\ &= -z^{j^{\mathrm{T}}}Lz^j + \tilde{U}^{j^{\mathrm{T}}}z^j \quad \text{(because } \dot{D}-2B \text{ is skew-symmetric)} \\ &\leqslant -\frac{2\lambda_{\min}(L)}{\lambda_{\max}(D)}W^j + \left(\frac{2}{\lambda_{\min}(D)}\right)^{1/2}\bar{W}^j|\tilde{U}^j(t)|\end{aligned}$$

where $\bar{W}^j \equiv (W^j)^{1/2}$. Dividing both sides of the above inequality by $\bar{W}^j(t)$ in the time interval where $W^j(t) \neq 0$, we obtain

$$\dot{\bar{W}}^j(t) \leqslant -a_0\,\bar{W}^j(t) + a_1|\tilde{U}^j(t)| \quad \text{(because } \dot{W}^j(t) = 2\bar{W}^j(t)\,\dot{\bar{W}}^j(t))$$

where $a_0 \equiv 2\lambda_{\min}(L)/\lambda_{\max}(D)$ and $a_1 \equiv (2/\lambda_{\min}(D))^{1/2}$. Hence, we have

$$\bar{W}^j(t) \leqslant e^{-a_0 t}\bar{W}^j(0) + a_1\int_0^t e^{-a_0(t-\tau)}|\tilde{U}^j(\tau)|\,\mathrm{d}\tau$$

which implies that z^j is $L_{pe}[0, t_f]$ stable whenever $\tilde{U}^j(t) \in L_{pe}[0, t_f]$ in the time interval where $W^j(t) \neq 0$ (Desoer and Vidyasagar 1975, Sastry and Bodson 1989). □

Appendix B: Proof of Theorem 1

From the definition of $\tilde{U}^j$ and the prediction learning rule

$$\begin{aligned}\Delta\tilde{U}^j &\equiv \tilde{U}^{j+1} - \tilde{U}^j = (T_d - H^{j+1}) - (T_d - H^j) \\ &= H^j - H^{j+1} = -\beta E^j = -\beta L z^j \end{aligned} \qquad \text{(B 1)}$$

Then, using the fact that $\dot{D}(q^j) - 2B(q^j, \dot{q}^j)$ is skew-symmetric and $D(q^j)$ and L are positive definite, we obtain

$$\begin{aligned}\Delta V^j &\equiv V^{j+1} - V^j \\ &= \int_0^t (\tilde{U}^{j+1^{\mathrm{T}}}L^{-1}\tilde{U}^{j+1} - \tilde{U}^{j^{\mathrm{T}}}L^{-1}\tilde{U}^j)\,\mathrm{d}\tau \\ &= \int_0^t (\Delta\tilde{U}^{j^{\mathrm{T}}}L^{-1}\Delta\tilde{U}^j + 2\Delta\tilde{U}^{j^{\mathrm{T}}}L^{-1}\tilde{U}^j)\,\mathrm{d}\tau \\ &= \int_0^t (\beta^2 z^{j^{\mathrm{T}}}Lz^j - 2\beta z^{j^{\mathrm{T}}}(D(q^j)\,\dot{z}^j + B(q^j,\dot{q}^j)\,z^j + Lz^j))\,\mathrm{d}\tau \\ &= -2\beta\int_0^t z^{j^{\mathrm{T}}}(D(q^j)\,\dot{z}^j + B(q^j,\dot{q}^j)\,z^j)\,\mathrm{d}\tau - \int_0^t (\beta(2-\beta)\,z^{j^{\mathrm{T}}}Lz^j)\,\mathrm{d}\tau \\ &= -\beta z^{j^{\mathrm{T}}}D(q^j)\,z^j + \beta\int_0^t z^{j^{\mathrm{T}}}(\dot{D}(q^j) - 2B(q^j,\dot{q}^j))\,z^j\,\mathrm{d}\tau - \int_0^t (\beta(2-\beta)\,z^{j^{\mathrm{T}}}Lz^j)\,\mathrm{d}\tau \\ &= -\beta z^{j^{\mathrm{T}}}D(q^j)\,z^j - \int_0^t (\beta(2-\beta)\,z^{j^{\mathrm{T}}}Lz^j)\,\mathrm{d}\tau \\ &\leqslant 0\end{aligned}$$

where $0 < \beta < 2$ and $z^j(0) = 0$. This proves (i) for all $t \in [0, t_f]$ and the equality holds

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only when $z^j = 0$. Note also that the tracking error $z^1 \in L_{2e}[0, t_f] \cap L_{\infty e}[0, t_f]$, since $\tilde{U}^1 = T_a \in L_{2e}[0, t_f] \cap L_{\infty e}[0, t_f]$. Then, the scalar function $V^1 \in L_{\infty e}[0, t_f]$, since

$$0 \leqslant V^1(t) = \int_0^t \tilde{U}^{1^{\mathrm{T}}}(\tau) L^{-1} \tilde{U}^1(\tau)\, \mathrm{d}\tau \leqslant \lambda_{\max}(L^{-1}) |T_a|_{2e}^2 < \infty$$

Since V^1 is bounded below, the monotonically non-increasing sequence $\{V^j\}$ converges to a fixed value. This implies $\Delta V^j \to 0$ and $z^j \to 0$ as $j \to \infty$, because $\Delta V^j \leqslant -\beta z^{j^{\mathrm{T}}} D(q^j) z^j \leqslant 0$. Since $z^j = \dot{e}^j + a e^j$ and $z^j \to 0$ for all $t \in [0, t_f]$, it follows that $e^j \to 0$ and $\dot{e}^j \to 0$ for all $t \in [0, t_f]$, which indicates (ii) and (iii). □

Appendix C: Proof of Corollary 2

Note in the proof of Theorem 1 that z^j satisfies

$$V^{j+1} + \beta z^{j^{\mathrm{T}}} D z^j + \int_0^t \beta z^{j^{\mathrm{T}}} (2-\beta) L z^j \, \mathrm{d}\tau \leqslant V^j$$

Since $0 < \beta < 2$, we obtain from this equation

$$\lambda_1 \beta z^{j^{\mathrm{T}}} z^j \leqslant \beta z^{j^{\mathrm{T}}} D z^j \leqslant V^j$$

and

$$\beta(2-\beta)\lambda_{\min}(L) \int_0^t z^{j^{\mathrm{T}}} z^j \, \mathrm{d}\tau \leqslant \int_0^t \beta z^{j^{\mathrm{T}}} (2-\beta) L z^j \, \mathrm{d}\tau \leqslant V^j$$

where $\lambda_1 I \leqslant D(q^j) \leqslant \lambda_2 I$. $z^j(t) \in L_{2e} \cap L_{\infty e}[0, t_f]$ follows immediately from these inequalities. Note also from the proof of Theorem 1 that $V^j \in L_{\infty e}[0, t_f]$ because $V^{j+1} \leqslant V^j \leqslant V^1 \in L_{\infty e}[0, t_f]$. Hence, $\tilde{U}^j \in L_{2e}[0, t_f]$ since

$$V^j(t) = \int_0^t \tilde{U}^{j^{\mathrm{T}}}(\tau) L^{-1} \tilde{U}^j(\tau)\, \mathrm{d}\tau \in L_{\infty e}[0, t_f]$$

On the other hand, since $\tilde{U}^{j+1} = T_a - \beta L \sum_{i=1}^{j} z^i$ from the prediction learning rule, it follows that $\tilde{U}^j \in L_{\infty e}[0, t_f]$. Moreover, using the error equation (6), it is trivial to prove that $\dot{z}^j(t) \in L_{\infty e}[0, t_f]$. Finally, if $\dot{T}_d \in L_{\infty e}[0, t_f]$, then, again from the prediction learning rule, we have $\dot{\tilde{U}}^{j+1} = \dot{T}_a - \beta L \sum_{i=1}^{j} \dot{z}^i$ from which $\dot{\tilde{U}}^j \in L_{\infty e}[0, t_f]$ follows from all $j \geqslant 1$. □

Appendix D: Proof of Theorem 2

The proof of the boundedness of tracking error z^1 and performance index V^1 is the same as in Theorem 1.

Applying the current learning rule to the system (5), we obtain

$$D(q^{j+1}) \dot{z}^{j+1} + B(q^{j+1}, \dot{q}^{j+1}) z^{j+1} + (1+\beta) L z^{j+1} = \tilde{U}^j \tag{D 1}$$

Note from the definition of $\tilde{U}^j$ and the current learning rule (8) that

$$\begin{aligned} \Delta \tilde{U}^{j+1} &\equiv \tilde{U}^{j+1} - \tilde{U}^j = (T_d - H^{j+1}) - (T_d - H^j) \\ &= H^j - H^{j+1} = -\beta E^{j+1} \end{aligned} \tag{D 2}$$

Then, as in the proof of Theorem 1, we obtain

$$\begin{aligned}\Delta V^{j+1} &\equiv V^{j+1}-V^{j}\\ &=\int_0^t(\tilde{U}^{j+1^{\mathrm{T}}}L^{-1}\tilde{U}^{j+1}-\tilde{U}^{j^{\mathrm{T}}}L^{-1}\tilde{U}^{j})\,\mathrm{d}\tau\\ &=\int_0^t(\Delta\tilde{U}^{j+1^{\mathrm{T}}}L^{-1}\Delta\tilde{U}^{j+1}+2\Delta U^{j+1^{\mathrm{T}}}L^{-1}\tilde{U}^{j})\,\mathrm{d}\tau\\ &=\int_0^t(\beta^2z^{j+1^{\mathrm{T}}}Lz^{j+1}-2\beta z^{j+1^{\mathrm{T}}}(D(q^{j+1})\dot{z}^{j+1}+B(q^{j+1},\dot{q}^{j+1})z^{j+1}+(1+\beta)Lz^{j+1}))\,\mathrm{d}\tau\\ &=-2\beta\int_0^t z^{j+1^{\mathrm{T}}}(D(q^{j+1})\dot{z}^{j+1}+B(q^{j+1},\dot{q}^{j+1})z^{j+1})\,\mathrm{d}\tau-\int_0^t\beta(2+\beta)z^{j+1^{\mathrm{T}}}Lz^{j+1}\,\mathrm{d}\tau\end{aligned}$$

The rest of the proof follows the same arguments as in Theorem 1. □

Appendix E: Proof of Lemma 2

Set a Lyapunov function candidate to

$$W^j=\tfrac{1}{2}z^{j^{\mathrm{T}}}Dz^j+\frac{1}{2\beta}\tilde{\theta}^{j^{\mathrm{T}}}\tilde{\theta}^j$$

Similar to Reed and Ioannou (1989), it is sufficient to consider the case when $|\hat{\theta}^1|>2\theta_0$ in examining the stability analysis. Applying the prediction learning rule and the prediction parameter estimator at $j=1$, we obtain along the error system trajectory (11)

$$\begin{aligned}\dot{W}^1&=z^{1^{\mathrm{T}}}D^1\dot{z}^1+\tfrac{1}{2}z^{1^{\mathrm{T}}}\dot{D}^1z^1+\frac{1}{\beta}\tilde{\theta}^{1^{\mathrm{T}}}\dot{\hat{\theta}}^1\\ &=z^{1^{\mathrm{T}}}(-Lz^1+Y_{\mathrm{e}}^1\tilde{\theta}^1+\tilde{U}^1)+\frac{1}{\beta}\tilde{\theta}^{1^{\mathrm{T}}}\dot{\hat{\theta}}^1\\ &=-z^{1^{\mathrm{T}}}Lz^1+z^{1^{\mathrm{T}}}(Y_{\mathrm{e}}^1\tilde{\theta}^1+Y_{\mathrm{d}}\tilde{\theta}^1+T_{\mathrm{a}})-\tilde{\theta}^{1^{\mathrm{T}}}(Y_{\mathrm{e}}^1+Y_{\mathrm{d}})^{\mathrm{T}}z^1-\frac{1}{\beta}\tilde{\theta}^{1^{\mathrm{T}}}\tilde{\theta}^1+\frac{1}{\beta}\tilde{\theta}^{1^{\mathrm{T}}}\theta\\ &=-x^{\mathrm{T}}K^*x+x^{\mathrm{T}}T_a^*\\ &\leqslant-\lambda_{\min}(K^*)|x|^2+|x|\,|T_{\mathrm{a}}^*|_m\end{aligned}$$

where

$$x=\begin{bmatrix}z^1\\ \tilde{\theta}^1\end{bmatrix},\quad K^*=\begin{bmatrix}L & 0\\ 0 & \frac{1}{\beta}I\end{bmatrix},\quad\text{and}\quad T_{\mathrm{a}}^*=\begin{bmatrix}T_{\mathrm{a}}\\ \frac{1}{\beta}\theta\end{bmatrix}$$

Then, $\dot{W}$ will be negative if

$$|x|\geqslant\frac{|T_{\mathrm{a}}^*|_m}{\lambda_{\min}(K^*)}$$

which implies that x is bounded. (See Reed and Ioannou 1989 and Lewis *et al.* 1993 for details.) Then $\tilde{\theta}^{1^{\mathrm{T}}}$ and z^1 are also bounded. □

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Appendix F: Proof of Theorem 3

Since $\hat{\theta}^1$ is bounded from the prediction parameter estimator in Lemma 2

$$\begin{aligned} V_a^1 &= \int_0^t (\tilde{U}^{1^T}(\tau) L^{-1} \tilde{U}^1(\tau) + \tilde{\theta}^{1^T}(\tau) S \tilde{\theta}^1(\tau)) \, d\tau \\ &= \int_0^t ((Y_d \tilde{\theta})^T L^{-1} (Y_d \tilde{\theta}^1 + T_a) + \tilde{\theta}^{1^T}(\tau) S \tilde{\theta}^1(\tau)) \, d\tau \\ &< \infty \text{ for all } t \in [0, t_f] \end{aligned}$$

Now, let $\Delta V_a^j \equiv V_a^{j+1} - V_a^j$, $\Delta \tilde{U}^j \equiv \tilde{U}^{j+1} - \tilde{U}^j$ and $\Delta \tilde{\theta}^j \equiv \tilde{\theta}^{j+1} - \tilde{\theta}^j$. From the error equation (11), the parameter estimator (12) and the prediction learning rule (B 1), it follows that

$$\begin{aligned} \Delta V_a^j &= V_a^{j+1} - V_a^j \\ &= \int_0^t (\tilde{U}^{j+1^T} L^{-1} \tilde{U}^{j+1} + \tilde{\theta}^{j+1^T} S \tilde{\theta}^{j+1} - \tilde{U}^{j^T} L^{-1} \tilde{U}^j - \tilde{\theta}^{j^T} S \tilde{\theta}^j) \, d\tau \\ &= \int_0^t (\Delta \tilde{U}^{j^T} L^{-1} \Delta \tilde{U}^j + 2 \Delta \tilde{U}^{j^T} L^{-1} \tilde{U}^j + \Delta \tilde{\theta}^{j^T} S \Delta \tilde{\theta}^j + 2 \Delta \tilde{\theta}^{j^T} S \tilde{\theta}^j) \, d\tau \\ &= \int_0^t (\beta^2 z^{j^T} L z^j - 2\beta z^{j^T} (D(q^j) \dot{z}^j + B(q^j, \dot{q}^j) z^j + L z^j - Y_e^j \tilde{\theta}^j)) \, d\tau \\ &\quad + \int_0^t (\beta^2 z^{j^T} (Y_e^j S^{-1} Y_e^{j^T}) z^j - 2\beta z^{j^T} Y_e^j \tilde{\theta}^j) \, d\tau \quad \text{(from (11))} \\ &= \int_0^t (\beta^2 z^{j^T} L z^j - 2\beta z^{j^T} (D(q^j) \dot{z}^j + B(q^j, \dot{q}^j) z^j + L z^j)) \, d\tau + \int_0^t \beta^2 z^{j^T} (Y_e^j S^{-1} Y_e^{j^T}) z^j \, d\tau \\ &= -2\beta \int_0^t z^{j^T} (D(q^j) \dot{z}^j + B(q^j, \dot{q}^j) z^j) \, d\tau - \beta \int_0^t z^{j^T} ((2 - \beta) L - \beta Y_e^j S^{-1} Y_e^{j^T}) z^j \, d\tau \end{aligned}$$

Integrating the first term by parts and exploiting the fact that $\dot{D} - 2B$ is skew-symmetric, we obtain

$$\Delta V_a^j = -\beta z^{j^T} D(q^j) z^j - \int_0^t (\beta z^{j^T} D_0^j z^j) \, d\tau \leqslant 0$$

where we have used the initial condition $z^j(0) = 0$. Hence, (i) follows for all $t \in [0, t_f]$. Following the similar argument as in Theorem 1, we have $\Delta V^j \to 0$ and $z^j \to 0$ for all $t \in [0, t_f]$, as $j \to \infty$. This implies (ii) and (iii). □

Appendix G: Proof of Theorem 4

As in Theorem 3, it is trivial to show that z^1 and V_a^1 are bounded. From equation (15), we have

$$\begin{aligned} \Delta \tilde{\theta}^{j+1} &\equiv \tilde{\theta}^{j+1} - \tilde{\theta}^j \\ &= -\beta S^{-1} Y_e^{j+1^T} z^{j+1} \end{aligned} \quad \text{(G 1)}$$

Then, using the error equation (14) and the current learning rules (8) and (15), we obtain

$$\begin{aligned}
\Delta V_a^{j+1} &\equiv V_a^{j+1} - V_a^j \\
&= \int_0^t (\Delta \tilde{U}^{j+1^T} L^{-1} \Delta \tilde{U}^{j+1} + 2\Delta \tilde{U}^{j+1^T} L^{-1} \tilde{U}^j + \Delta \tilde{\theta}^{j+1^T} S \Delta \tilde{\theta}^{j+1^T} + 2\Delta \tilde{\theta}^{j+1} S \tilde{\theta}^j)\, d\tau \\
&= \int_0^t (\beta^2 z^{j+1^T} L z^{j+1} - 2\beta z^{j+1^T} (D(q^{j+1}) \dot{z}^{j+1} + B(q^{j+1}, \dot{q}^{j+1}) z^{j+1} \\
&\quad + (1+\beta) L z^{j+1} - Y_e^{j+1} \tilde{\theta}^j + \beta Y_e^{j+1} S^{-1} Y_e^{j+1^T} z^{j+1}))\, d\tau \\
&\quad + \int_0^t (\beta^2 z^{j+1^T} (Y_e^{j+1} S^{-1} Y_e^{j+1^T}) z^{j+1} - 2\beta z^{j+1^T} Y_e^{j+1} \tilde{\theta}^j)\, d\tau \\
&= \int_0^t (\beta^2 z^{j+1^T} L z^{j+1} - 2\beta z^{j+1^T} (D(q^{j+1}) \dot{z}^{j+1} + B(q^{j+1}, \dot{q}^{j+1}) z^{j+1} + (1+\beta) L z^{j+1}))\, d\tau \\
&\quad - \int_0^t \beta^2 z^{j+1^T} (Y_e^{j+1} S^{-1} Y_e^{j+1^T}) z^{j+1}\, d\tau \\
&= -2\beta \int_0^t z^{j+1^T} (D(q^{j+1}) \dot{z}^{j+1} + B(q^{j+1}, \dot{q}^{j+1}) z^{j+1})\, d\tau \\
&\quad - \beta \int_0^t z^{j+1^T} ((2+\beta) L + \beta Y_e^{j+1} S^{-1} Y_e^{j+1^T}) z^{j+1}\, d\tau \\
&= -z^{j+1^T} D(q^{j+1}) z^{j+1} + \beta \int_0^t z^{j+1^T} (\dot{D}(q^{j+1}) z^{j+1} - 2B(q^{j+1}, \dot{q}^{j+1})) z^{j+1}\, d\tau \\
&\quad - \beta \int_0^t z^{j+1^T} ((2+\beta) L + \beta Y_e^{j+1} S^{-1} Y_e^{j+1^T}) z^{j+1}\, d\tau \\
&= -z^{j+1^T} D(q^{j+1}) z^{j+1} - \beta \int_0^t z^{j+1^T} ((2+\beta) L + \beta Y_e^{j+1} S^{-1} Y_e^{j+1^T}) z^{j+1}\, d\tau \\
&\quad \text{(because } \dot{D}(q^{j+1}) - 2B(q^{j+1}, \dot{q}^{j+1}) \text{ is skew-symmetric)} \\
&\leqslant 0
\end{aligned}$$

(because $D(q^{j+1})$ and L are positive definite and $Y_e^{j+1} S^{-1} Y_e^{j+1^T}$ is non-negative definite).

The rest of the proof follows the same arguments as in Theorem 3. □

Appendix H: Proof of Theorem 5

The prediction parameter estimator (12) states that

$$\begin{aligned}
\tilde{\theta}^j &= \theta - \hat{\theta}^j \\
&= \theta - \hat{\theta}^{j+1} + \beta S^{-1} Y_e^{j^T} z^j \\
&= \tilde{\theta}^{j+1} + \beta S^{-1} Y_e^{j^T}
\end{aligned}$$

and

$$\tilde{\theta}^{j+n} = \tilde{\theta}^{j+N+1} + \beta \sum_{i=j+n}^{j+N} S^{-1} Y_e^{i^T} z^i \tag{H 1}$$

for all $1 \leqslant n \leqslant N$.

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Multiplying Y_e^{j+n-1} on both sides of the above parameter estimator at the $(j+n-1)$th iteration, we obtain

$$Y_e^{(j+n-1)}\tilde{\theta}^{(j+n)} = Y_e^{(j+n-1)}\tilde{\theta}^{(j+n-1)} - \beta Y_e^{(j+n-1)} S^{-1} Y_e^{(j+n-1)} z^{(j+n-1)} \tag{H 2}$$

Now, define

$$S_N^{n-1} \equiv Y_e^{(j+n-1)}\tilde{\theta}^{(j+n-1)} - \beta \sum_{i=j+n-1}^{j+N} Y_e^{(j+n-1)} S^{-1} Y_e^{i^{\mathrm{T}}} z^i$$

for $1 \leqslant B+1$. Then, from the convergence results established in Theorem 3, it is trivial to prove that for $1 \leqslant n \leqslant N+1$

$$\lim_{j\to\infty} S_N^{n-1} = 0 \quad \text{for all } t \in [0, t_f] \tag{H 3}$$

On the other hand, in view of (H 1) and (H 2), S_N^{n-1} becomes

$$S_N^{n-1} = Y_e^{(j+n-1)}\tilde{\theta}^{(j+N+1)} \quad \text{for } 1 \leqslant n \leqslant N+1 \tag{H 4}$$

Hence, if we define S_N as $S_N \equiv \sum_{n=1}^{N+1} S_N^{(n-1)^{\mathrm{T}}} S_N^{(n-1)}$, then from (H 4), we obtain

$$S_N = \tilde{\theta}^{(j+N+1)^{\mathrm{T}}} \sum_{i=j}^{j+N} (Y_e^{i^{\mathrm{T}}} Y_e^i)\tilde{\theta}^{(j+N+1)} \tag{H 5}$$

Applying the PE condition of Y_e^j to (H 5) we deduce that

$$0 \leqslant \alpha_1 \tilde{\theta}^{(j+N+1)^{\mathrm{T}}}\tilde{\theta}^{(j+N+1)} \leqslant S_N \leqslant \alpha_2 \tilde{\theta}^{(j+N+1)^{\mathrm{T}}}\tilde{\theta}^{(j+N+1)} \tag{H 6}$$

Combining (H 6) with (H 3), we have $\lim_{j\to\infty}\tilde{\theta}^j = 0$.

The convergence of the current parameter estimator (15) can be established in a similar fashion. □

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