Adaptive Fuzzy Learning Control for a Class of Nonlinear Dynamic Systems

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This paper presents an adaptive iterative learning control scheme that is applicable to a class of nonlinear systems. The control scheme guarantees system stability and boundedness by using the feedback controller coupled with the fuzzy compensator and achieves precise tracking by using the iterative learning rules. In the feedback plus fuzzy compensator unit, the feedback control part stabilizes the overall closed-loop system and keeps its error bounded, and the fuzzy compensator estimates and compensates for the nonlinear part of the system, thereby keeping the feedback gains reasonably low in the feedback controller. The fuzzy compensator is designed by applying the fuzzy approximation technique to the uncertain nonlinear term to be compensated. In the iterative learning controller, a simple learning control rule is used to achieve precise tracking of the reference signal and a parameter learning algorithm is used to update the parameters in the fuzzy compensator so as to identify the uncertain nonlinearity as much as possible.

1. INTRODUCTION

There has been a substantial amount of research on the development of various iterative learning control methods. The concept of iterative learning control has been introduced to improve tracking performance in an iterative manner by attempting to execute the desired motion repeatedly. The control effort in each attempt is improved by using the tracking error signals obtained from previous trial. Arimoto et al.[1] proposed a general iterative learning control method for a class of nonlinear systems whose input and output gain matrices are of linear time-invariant form. In their scheme, the time derivative of the current output error is used to update the learning control input for the next iteration. Ahn et al.[2] and Jang et al.[3] proposed iterative learning control laws using the relative degree concept of nonlinear system introduced by Isidori.[4] Their controllers still use derivatives of output errors and their convergence

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conditions are implicitly coupled with the decoupling matrix of the nonlinear system. Messner et al.[5] introduced an adaptive learning rule based on the linear integral equations: they represent the unknown robot dynamics with a known nondegenerate symmetric Hilbert−Schmidt kernel and an unknown influence function that is to be updated by the learning rule. Chien and Liu[6] presented the P-type robust iterative learning controller for nonlinear systems where the product of input gain matrix and partial derivative of output gain is positive definite. Amaiih et al.[7] proposed an iterative learning control technique for linear systems using optimal feedback and feedforward actions. Park et al.[8] presented a simple and efficient adaptive learning control law for robot systems based on the linear parameterization technique. In their control scheme, they updated not only the learning control input but also the parameter estimates. This control scheme, however, cannot be applied to general nonlinear systems because they cannot be described as a convenient linear parameterization form in general. To overcome this problem, a fuzzy logic representation technique is introduced in this paper where the nonlinear compensation term is approximated by a fuzzy logic system that can be formulated as a linear parameterization form using the fuzzy basis function approach.

The research on the fuzzy logic control[9] has been motivated by Ref. 10, which describes the linguistic approach to the system analysis using the theory of fuzzy sets. Fuzzy logic, which forms the basis of fuzzy control methods, is similar to human thinking and natural language representation and provides an effective means of capturing approximated and inexact nature of the real world. Lee[11] showed that from the mathematical point of view, a fuzzy logic system can in fact be regarded as a mapping from the input space to the output space that approximates the nonlinear function within a given accuracy. It describes the nonlinear function by using the linguistic descriptions obtained either from the experts or from the experimental data and uses the fuzzy inference technique to obtain the results. Wang[12] introduced an universal approximation theory for a class of fuzzy logic systems: any continuous or L_2 nonlinear function can be approximated with arbitrary accuracy by a fuzzy logic system. This fact was extensively used in his other paper[13] to develop adaptive fuzzy control methods for a class of nonlinear systems. Since the universal approximation theory holds only in the compact set, he added yet another supervisory control term to keep the state of the system within the compact set. Su and Stepanenko[14] presented an adaptive fuzzy controller with switching control technique and proved that the system errors converge to zero. Spooner and Passino[15] developed a stable adaptive controller using fuzzy systems and neural networks, in which they used output error indirect adaptive control structure based on feedback linearization.

Applying the fuzzy logic technique to the iterative learning controller, we present an adaptive fuzzy learning control scheme in this paper that is applicable to a broad class of nonlinear dynamic systems. The proposed controller is composed of three blocks: feedback control block, adaptive fuzzy control block, and learning control block. The linear feedback control plus adaptive fuzzy control block stabilizes the overall closed loop system and keeps its errors within uniform bound. Particularly, the adaptive fuzzy control block compensates for the nonlinear part of the system, thereby reducing the load from the feedback control block and keeping the feedback gain reasonably small. It also helps relax the conditions that are normally required to prove the error convergence of the learning control system.[8] The learning control block updates the input signals by using the feedback signals obtained from linear feedback control block. In contrast with many other learning control schemes,[2,3] the scheme presented updates iteratively not only the learning control signals but also the parameters in the fuzzy logic system based on the fuzzy basis function approach. In addition, the learning control algorithm achieves precise tracking without using any output derivative terms that are vulnerable to noise.

The paper is organized as follows. Section 2 formulates the problem and Section 3 introduces the fuzzy logic description of nonlinear systems. Section 4 presents a fuzzy learning controller without parameter adaptation and shows that the tracking error converges to zero. Section 5 presents a fuzzy learning controller with parameter adaptation along with its convergence properties. Section 6 shows the simulation results and Section 7 makes conclusions.

The following notations and definitions will be used throughout the paper. R represents the scalar real space endowed with the L_∞ norm $\|x(t)\| = \sup_{t \in [0, t_f]} |x(t)|$. R^n represents the n-dimensional vector space over R endowed with the L_∞ norm $\|\mathbf{x}(t)\| = \sup_{t \in [0, t_f]} |\mathbf{x}(t)|$, where $|\cdot|$ denotes the Euclidean norm. The nth derivative of $x(t)$ with respect to time t is denoted as $x^{(n)}(t)$.

2. PROBLEM FORMULATION

The nonlinear dynamic systems that are considered in this section are of the form,

$$g(\mathbf{x}(t))x^{(n)}(t) + f(\mathbf{x}(t)) = u(t) \tag{1}$$

where $\mathbf{x}(t) = (x(t), \dot{x}(t), \dots, x^{(n-1)}(t)) \in R^n$ is the state of the system and f: $R^n \to R$ and g: $R^n \to R$. Note that a large class of nonlinear systems that include robotic systems belong to or can be transformed to the form (1). Note that, by dividing (1) by $g(x(t))$, we have

$$x^{(n)}(t) = p(\mathbf{x}(t)) + q(\mathbf{x}(t))u(t)$$

which is known as normal form. For subsequent development, the following assumptions that hold in many practical systems are imposed on system (1).

ASSUMPTION 1. *f: $R^n \to R$ and g: $R^n \to R$ are Lipshitz on R^n That is, there exists $l_g, l_f > 0$ such that*

$$|g(\mathbf{x}_2(t)) - g(\mathbf{x}_1(t))| \le l_g |\mathbf{x}_2(t) - \mathbf{x}_1(t)|$$

$$|f(\mathbf{x}_2(t)) - f(\mathbf{x}_1(t))| \le l_f |\mathbf{x}_2(t) - \mathbf{x}_1(t)|$$

for all $\mathbf{x}_1(t), \mathbf{x}_2(t) \in R^n$.

ASSUMPTION 2. *For all* $\mathbf{x}(t) \in U$ *where* U *is a compact subset of* R^n, *it holds that*

$$g_L \leq g(\mathbf{x}(t)) \leq g^U \qquad |\dot{g}(\mathbf{x}(t))| \leq \alpha_g(t)$$

for some $g_L, g^U > 0$ *and* $\alpha_g(t) \geq 0$.

Remark 1. The assumption that $|\dot{g}(\mathbf{x}(t))| \leq \alpha_g(t)$ for some $\alpha_g(t) \geq 0$ is satisfied for a large class of nonlinear systems. For a robot dynamics example, $\alpha_g(t)$ can be chosen as $c|\mathbf{x}(t)|$ for some $c > 0$ from the fact that $g(\mathbf{x}(t)) = g(x^{(n-2)}, x^{(n-2)}(t), \ldots, \dot{x}(t), x(t))$.

ASSUMPTION 3. *The desired trajectory* $x_d(t)$ *is continuously differentiable up to the nth order, and there exists an upper bound of* $u_f d(t)$ *such that* $|u_d(t)| \leq u_d^b$ *for all* $t \in [0, t_f]$. *Also,* $\mathbf{x}(0) = \mathbf{x}_d(0)$ *is assumed for simplicity.*

Under these assumptions, the learning control problem can be formulated as follows:

Problem Statement. Suppose that $\mathbf{x}(t) \in R^n$ is measurable for $t \in [0, t_f]$ and is in the interior of a domain $U \subset R^n$, which is compact and simply connected. Then find a sequence of piecewise continuous control input $u^j(t) \in R$ for uncertain system (4) with which the system trajectory $\mathbf{x}^j(t)$ converges to $\mathbf{x}_d(t)$. In other words, given $\epsilon > 0$, find a sequence of $u^j(t)$ such that there exists a positive integer $N > 0$ such that

$$\sup_{t \in [0, t_f]} |\mathbf{x}_d(t) - \mathbf{x}^j(t)| \leq \epsilon \qquad \text{for all } j > N$$

Our approach to the above problem is to approximate the nonlinear functions $f(\mathbf{x}(t))$ and $g(\mathbf{x}(t))$ with the fuzzy logic systems and use them to build a fuzzy compensator which forms one of the components in the fuzzy learning controller.

3. FUZZY LOGIC DESCRIPTION OF NONLINEAR SYSTEMS

As stated in the universal approximation' theorem as in Refs. 12 and 13, any continuous nonlinear system can be approximated by a fuzzy logic system with arbitrary accuracy in the compact domain. Thus, the nonlinear functions $g(\mathbf{x}(t))$ and $f(\mathbf{x}(t))$ in (1) can be approximated by the fuzzy logic systems $\hat{g}(\mathbf{x}(t)|\theta_g^*)$ and $\hat{f}(\mathbf{x}(t)|\theta_f^*)$ as follows:

$$\begin{aligned} g(\mathbf{x}(t)) &= \hat{g}(\mathbf{x}(t)|\theta_g^*) + w_g(\mathbf{x}(t)) = \zeta_g^T(\mathbf{x}(t))\,\theta_g^* + w_g(\mathbf{x}(t)) \qquad \text{and} \\ f(\mathbf{x}(t)) &= \hat{f}(\mathbf{x}(t)|\theta_f^*) + w_f(\mathbf{x}(t)) = \zeta_f^T(\mathbf{x}(t))\,\theta_f^* + w_f(\mathbf{x}(t)) \end{aligned} \tag{2}$$

where $\zeta_g(\mathbf{x}(t))$ and $\zeta_f(\mathbf{x}(t))$ are, respectively, the fuzzy basis function vectors of $\hat{g}(\mathbf{x}(t)|\theta_g^*)$ and $\hat{f}(\mathbf{x}(t)|\theta_f^*)$ and $w_g(\mathbf{x}(t))$ and $w_f(\mathbf{x}(t))$ are the corresponding fuzzy approximation errors. Here the optimum parameter vectors θ_g^* and θ_f^* are defined by

$$\theta_g^* = \operatorname{Arg}\left[\min_{\theta_g \in \Omega_g}\left(\sup_{\mathbf{x}(t)\in U}\left|g(\mathbf{x}(t)) - \hat{g}\big(\mathbf{x}(t)|\theta_g\big)\right|\right)\right] \qquad \text{and}$$

$$\theta_f^* = \operatorname{Arg}\left[\min_{\theta_f \in \Omega_f}\left(\sup_{\mathbf{x}(t)\in U}\left|f(\mathbf{x}(t)) - \hat{f}\big(\mathbf{x}(t)|\theta_f\big)\right|\right)\right]$$

where Ω_g and Ω_f are some bounded feasible sets of θ_g and θ_f respectively. Therefore, it is obvious that $|\theta_f^*| \le \theta_f^b$ and $|\theta_g^*| \le \theta_g^b$ for some $\theta_f^b, \theta_g^b > 0$. For notational brevity, we will use in the following $\mathbf{x}$ for $\mathbf{x}(t)$, g for $g(\mathbf{x}(t))$, f for $f(\mathbf{x}(t))$, g^j for $g(\mathbf{x}^j(t))$, f^j for $f(\mathbf{x}^j(t))$, g_d for $g(\mathbf{x}_d(t))$, f_d for $f(\mathbf{x}_d(t))$, $\hat{g}^*$ for $\hat{g}(\mathbf{x}(t)|\theta_g^*)$, $\hat{f}^*$ for $\hat{f}(\mathbf{x}(t)|\theta_f^*)$, $\hat{g}^{*j}$ for $\hat{g}(\mathbf{x}^j(t)|\theta_g^*)$, $\hat{f}^{*j}$ for $\hat{f}(\mathbf{x}^j(t)|\theta_f^*)$, $\hat{g}_d^*$ for $\hat{g}(\mathbf{x}_d(t)|\theta_g^*)$, $\hat{f}_d^*$ for $\hat{f}(\mathbf{x}_d(t)|\theta_f^*)$, w_g for $w_g(\mathbf{x}(t))$, w_f for $w_f(\mathbf{x}(t))$, w_g^j for $w_g(\mathbf{x}^j(t))$, w_f^j for $w_f(\mathbf{x}^j(t))$, w_{g_d} for $w_g(\mathbf{x}_d(t))$, and w_{f_d} for $w_f(\mathbf{x}_d(t))$.

Now, let $\mathbf{x}^j$ and $\mathbf{x}^d$ be, respectively, the resulting states of (1) due to inputs u^j and u_d. Let us also define $e^j = x^j - x_d$ and $z^j = (a + (d/dt))^{n-1} e^j = \mathbf{k}^T\mathbf{e}^j$, where $\mathbf{k} = (k_{n-1}, k_{n-2}, \ldots, k_1, k_0)^T$, $\mathbf{e}^j = (e^{j^{(n-1)}}, e^{j^{(n-2)}}, \ldots, \dot{e}^j, e^j)^T$, and $k_i = {}_{n-1}C_i \cdot a^{n-1-i} = ((n-1)!/(n-1-i)!i!) \cdot a^{n-1-i}$ for all $1 \le i \le n-1$. The design parameter $\mathbf{k}$ is chosen in such a way that $k_{n-1}s^{n-1} + k_{n-2}s^{n-2} + k_1 s + k_0$ is Hurwitz. The auxiliary signal z^j can be rewritten as $z^j = e^{j^{n-1}} + \bar{\mathbf{k}}^T\bar{\mathbf{e}}^j$ where $\bar{\mathbf{k}} = (k_{n-2}, k_{n-3}, \ldots, k_1, k_0)$ and $\bar{\mathbf{e}}^j = (e^{j^{(n-2)}}, e^{j^{(n-3)}}, \ldots, \dot{e}^j, e^j)$. Then it follows from (1) that

$$\begin{aligned} g^j\big(x^{j^{(n)}} - x_d^{(n)}\big) &+ \big(g^j - g_d\big)x_d^{(n)} + f^j - f_d \\ &= g^j\dot{z}^j - g^j\bar{\mathbf{k}}^T\dot{\bar{\mathbf{e}}}^j + \big(g^j - g_d\big)x_d^{(n)} + f^j - f_d \\ &= u^j - u_d \end{aligned} \tag{3}$$

Substituting (2) into (3), we have

$$\begin{aligned} g^j\dot{z}^j - g^j\bar{\mathbf{k}}^T\dot{\bar{\mathbf{e}}}^j &+ \big(\hat{g}^{*j} - \hat{g}_d^*\big)x_d^{(n)} + \big(w_g^j - w_{g_d}\big)x_d^{(n)} \\ &+ \hat{f}^{*j} - \hat{f}_d^* + \big(w_f^j - w_{f_d}\big) = u^j - u_d \end{aligned}$$

which can be rearranged further to

$$\begin{aligned} g^j\dot{z}^j &+ \big(\hat{g}^{*j} - \hat{g}_d^*\big)x_d^{(n)} + \hat{f}^{*j} - \hat{f}_d^* \\ &= u^j - u_d - \big(w_g^j - w_{g_d}\big)x_d^{(n)} - \big(w_f^j - w_{f_d}\big) + g^j\bar{\mathbf{k}}^T\dot{\bar{\mathbf{e}}}^j \end{aligned} \tag{4}$$

By using the fuzzy basis function representation in (2), Eq. (4) can be reduced to

$$g^j\dot{z}^j + Y_s^j\theta^* = u^j - u_d - w_e^j \tag{5}$$

where $Y_s^j = (\zeta_g^T(\mathbf{x}^j)x_d^{(n)} - \zeta_g^T(\mathbf{x}_d)x_d^{(n)}, \zeta_f^T(\mathbf{x}^j) - \zeta_f^T(\mathbf{x}_d))$, $\theta^{*T} = (\theta_g^{*T}, \theta_f^{*T})$, and $w_e^j = (w_g^j - w_{g_d})x_d^{(n)} + (w_f^j - w_{f_d}) - g^j\bar{\mathbf{k}}^T\dot{\mathbf{e}}^j$. Note here that (5) is in the linear form with respect to the parameter θ^* with additional fuzzy approximation error w_e^j. This fact will be used extensively in the following sections to prove stability and convergence of the closed loop system. Now, the combined fuzzy approximation error w_e^j satisfies the following lemma.

LEMMA 1. *The fuzzy approximation error w_e^j in (5) satisfies*

$$|w_e^j| \le l_w |\mathbf{x}^j - \mathbf{x}_d| = l_w \sum_{i=0}^{n-1} \left| \frac{d^i e^j}{dt^i} \right| \tag{6}$$

for some $l_w > 0$ and for all $j \ge 1$.

Proof. See Appendix A. ■

Remark 2. Because we have to calculate $(\partial\zeta_g^T(\mathbf{x})/\partial\mathbf{x})$ and $(\partial\zeta_f^T(\mathbf{x})/\partial\mathbf{x})$ to compute l_w, the fuzzy membership functions employed to construct $\zeta_g(\mathbf{x})$ and $\zeta_f(\mathbf{x})$ must be differentiable. Thus the differentiable membership functions such as Gausszian-type membership functions are required in building the fuzzy logic systems.

4. FUZZY LEARNING CONTROL

By using the fuzzy system representation (5) of nonlinear dynamic system (1), we design a fuzzy learning control strategy in this section with the assumption that the parameter θ^* is fully known. The next section deals with the case when θ^* is not completely known. The fuzzy learning control law that we propose to use for system (1) is as follows:

$$u^j = c^j + u_f^j + h^j \tag{7}$$

where

$$\begin{aligned} c^j &= Y_s^j\theta^* \qquad u_f^j = -\big(\beta_1 + k_g\alpha_g(t)\big)z^j \\ h^j &= \mathrm{proj}(\bar{h}^j) \qquad \bar{h}^j = h^{j-1} - \beta_2 z^{j-1} \end{aligned} \tag{8}$$

where $k_g \ge \frac{1}{2}$, $\beta_1 > 0$, $\beta_2 > 0$, and proj $(\bar{h}^j)$ is the projection operator defined by

$$\mathrm{proj}(\bar{h}^j) = \begin{cases} u_d^b & \text{if } \bar{h}^j \ge u_d^b \\ -u_d^b & \text{if } \bar{h}^j \le -u_d^b \\ \bar{h}^j & \text{otherwise} \end{cases}$$

The feedback term u_f^j with nonlinear compensation term c^j stabilizes the overall closed loop system. Note here that c^j is an approximated fuzzy logic representation of the nonlinear term $(g^j - g_d)x_d^{(n)} + (f^j - f_d)$. Its main effect is to reduce the control load from the feedback term u_f^i and maintain small feedback gain β_1. h^j is a learning rule that estimates and compensates for the desired control input u_d. The projection operator guarantees that h^j is bounded. Substituting (7) into (5), we have

$$g^j \dot{z}^j + \beta_1 z^j + k_g \alpha_g(t) z^j = \tilde{u}^j - w_e^j \tag{9}$$

where $\tilde{u}^j = h^j - u_d$.

For the proof of Theorem 1 and 2, the following lemmas are introduced

LEMMA 2. *Let $e_n(t) = ((d/dt) + a)^n e(t)$ and $e_i(t) = ((d/dt) + a)^i e(t)$ where n is a positive integer and $0 \le i \le n$. Then*

$$\int_0^t e_n^2(\tau)\, d\tau = \int_0^t \sum_{i=0}^{n} {}_nC_i \left(\frac{d^i e(\tau)}{d\tau^i} \right)^2 a^{2(n-i)}\, d\tau + a \sum_{k=0}^{n-1} \sum_{i=0}^{k} {}_kC_i \left(\frac{d^i e_{n-k-1}(t)}{d\tau^i} \right)^2 a^{2(k-i)} \tag{10}$$

Proof. See Appendix B. ■

Another useful lemma is given as

LEMMA 3. *Given the auxiliary signal z^j and w_e^j as in (6), there exists a symmetric positive definite matrix Q such that*

$$\int_0^t \left(-\beta_h |z^j|^2 + l_w \sum_{i=0}^{n-1} |e^{j(i)}| \cdot |z^j| \right) d\tau \le -\int_0^t \left(\mathbf{e}^{j^T} Q \mathbf{e}^j \right) d\tau \tag{11}$$

where

$$Q = Q^T = \begin{pmatrix} q_{n-1,n-1} & q_{n-1,n-2} & \cdots & q_{n-1,0} \\ q_{n-2,n-1} & q_{n-2,n-2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ q_{0,n-1} & \cdots & \cdots & q_{0,0} \end{pmatrix} \ge 0$$

and

$$q_{l,m} = q_{m,l} = \begin{cases} -\dfrac{1}{2} l_w (k'_l + k'_m) & l \neq m \\ \beta_h k_l'^2 - l_w k'_m & l = m \end{cases} \tag{12}$$

where $k'_l = k_l a^{(n-1-l)} = {}_{n-1}C_l \cdot a^{2(n-1-l)}$.

Proof. Using (6) and Lemma 2, we have

$$\begin{aligned} &\int_0^t \left(-\beta_h |z^j|^2 + l_w \sum_{i=0}^{n-1} |e^{j^{(i)}}| \cdot |z^j| \right) d\tau \\ &\leq -\int_0^t \left(\beta_h \sum_{i=0}^{n-1} k'_i |e^{j^{(i)}}|^2 - l_w \sum_{k=0}^{n-1} \sum_{i=0}^{n-1} k_i |e^{j^{(i)}}| |e^{j^{(k)}}| \right) d\tau \\ &\leq -\int_0^t \mathbf{e}^{j^T} Q \mathbf{e}^j d\tau \end{aligned} \tag{13}$$

Note here that $Q \geq 0$ whenever sufficiently large values are chosen for $\beta_h > 0$. ■

Now, we propose a learning controller that keeps the tracking errors bounded and drives them to zero.

THEOREM 1. *Let* β_1, β_2 *satisfy* $\beta_1 = \frac{1}{2}(\beta_2 + 2\beta_h)$, *where* β_h, $\mathbf{k}$ *are chosen such that* Q *defined in* (12) *satisfies* $Q = Q^T \geq 0$. *Then the fuzzy learning control system* (9) *is bounded as*

$$|z^j(t)| \leq \left(\frac{1}{\beta_2 g_L} v^1(t) \right)^{1/2}$$

and converges as follows.:

(i) $\lim_{j \to \infty} v^j(t) = v(t)$

(ii) $\lim_{j \to \infty} v^j(t) = 0$ *for all* $t \in [0, t_f]$

where $v^j(t) = \int_0^t \tilde{u}^{j^2}(\tau)\, d\tau$ *for all* $t \in [0, t_f]$ *and for all* $j \geq 1$.

Proof. See Appendix C. ■

Although the feedback gain β_h depends on the unknown constant l_w as in (11), it is not more restrictive than many other learning controllers or adaptive fuzzy controllers.[11,15,16] Further, it decreases as the fuzzy rules increase.[17] Since

z^j converges to zero and $e^{j^{(i)}}(0) = 0$ for $1 \le i \le n-1$, $e^{j^{(i)}}$ for $1 \le i \le n-1$ also converges to zero. Note that v^1 decreases as feedback gain β_2 increases. Thus, we can keep the state $\mathbf{x}^j$ within a compact set U by increasing the feedback gain β_2. This fact is very important in fuzzy logic systems because the universal approximation theorem holds only when the states are within a compact set. Wang[12] used supervisory control action in his controller to guarantee that the states are bounded. The supervisory control action, however, may cause high frequency chattering with large amplitude. In establishing that the states are bounded, our proof is much simpler than that of Kuc et al.[18]

5. ADAPTIVE FUZZY LEARNING CONTROL

Since the optimum parameter vector θ^* is not known in general, we need an additional learning algorithm that searches for the optimum θ^* and approximates the nonlinear term $(g^j - g_d)x_d^{(n)} + f^3 - f_d$ as closely as possible. Let us first modify the feedback control u_f^j as

$$u_f^j = -\left(\beta_1 + k_g \alpha_g(t) + Y_s^j Y_s^{j^T} \right) z^j \tag{14}$$

Since the parameter vector θ^* is not known, the estimated parameter $\hat{\theta}^j$ is used instead in computing the adaptive fuzzy control input c^j:

$$c^j = Y_s^j \hat{\theta}^j \tag{15}$$

Substituting (14) and (15) into (7) and then (7) into (5), we have

$$g^j \dot{z}^j + \beta_1 z^j + k_g \alpha_g(t) z^j + Y_s^j Y_s^{j^T} z^j = Y_s^j \tilde{\theta}^j + \tilde{u}^j - w_e^j \tag{16}$$

where $\tilde{\theta}^j = \hat{\theta}^j - \theta^*$. At this point, we propose to use a parameter learning rule for $\hat{\theta}^j$:

$$\begin{aligned} \hat{\theta}^{j+1^T} &= \text{proj}\{\hat{\bar{\theta}}^{j+1}\} = \left\{ \text{proj}\left(\hat{\bar{\theta}}_1^{j+1}\right), \ldots, \text{proj}\left(\hat{\bar{\theta}}_N^{j+1}\right) \right\} \quad \text{and} \\ \hat{\bar{\theta}}^{j+1} &= \hat{\theta}^j - \beta_3 Y_s^{j^T} z^j \end{aligned} \tag{17}$$

where $\beta_3 > 0$ and

$$\text{prob}\left(\hat{\bar{\theta}}_i^{j+1}\right) = \begin{cases} \theta^b & \text{if } \hat{\bar{\theta}}_i^{j+1} \ge \theta^b \\ -\theta^b & \text{if } \hat{\bar{\theta}}_i^{j+1} \ge \theta^b \\ \hat{\bar{\theta}}_i^{j+1} & \text{otherwise} \end{cases}$$

Then the following theorem establishes that the fuzzy learning control system (14), (15), and (17) with the input learning rule as in (8) keeps the tracking errors bounded and drives them to zero.

THEOREM 2. *Let $\beta_1, \beta_2, \beta_3$ satisfy $\beta_1 = \frac{1}{2}(\beta_2 + \beta_h)$, $\beta_2 = \rho\beta_3$ for some $\rho > 0$ and $0 \le \beta_3 \le 2$. Also, choose β_h and $\mathbf{k}$ in such a way that $Q = Q^T \ge 0$, where Q is defined in Theorem.* 1. *Then, the adaptive fuzzy learning controller* (7) *with the learning control rule* (8) *and the parameter learning rule* (17) *for the uncertain dynamic system* (16) *is bounded as*

$$|z^j(t)| \le \left(\frac{1}{\beta_2 g_L} v_a^1(t) \right)^{1/2}$$

and converges as follows:

(i) $\lim_{j\to\infty} v_a^j(t) = v_a(t)$
(ii) $\lim_{j\to\infty} z^j(t) = 0$ *for all* $t \in [0, t_f]$

where $v_a^j(t) = \int_0^t (\tilde{u}^{j^2}(\tau) + \rho\tilde{\theta}^{j^T}(\tau)\tilde{\theta}^j(\tau))\, d\tau$ *for all* $t \in [0, t_f]$ *and* $j \ge 1$.

Proof. See Appendix D. ■

Note here that v_a^1 decreases as feedback gain β_2 increases or as $\hat{\theta}^j$ comes closer to θ^*. Thus, we can keep the state $\mathbf{x}^j$ within the preset compact set U by increasing the feedback gain β_2 or by choosing $\hat{\theta}^j$ appropriately. The fuzzy compensator is particularly useful when some information on the system is available, since it can be exploited in implementing the fuzzy membership functions and initial parameter $\theta(0)$.

Remark 3. Note that ρ is a weighting coefficient between u_h^j and u_c^j. If there is little information available on the system, ρ must be set greater than 1 to satisfy $\beta_2 > \beta_3$. In fact, the fuzzy compensator starting with little information on the system may act as a disturbance to the system. So the condition $\beta_2 > \beta_3$ is required to prevent the negative effect from the fuzzy compensator from happening.

6. SIMULATION RESULTS

The proposed controller is applied in this simulation test to the inverted pendulum model, whose dynamics are given as

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = \frac{g \sin x_1(t) - \dfrac{ml x_2^2(t) \cos x_1(t) \sin x_1(t)}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1(t)}{m_c + m}\right)} + \frac{\dfrac{\cos x_1(t)}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1(t)}{m_c + m}\right)} u \tag{18}$$

where $g = 9.8$ m/s is the acceleration due to gravity, m_c is the mass of the cart, m is the mass of the pole, and l is the half-length of the pole. The state $(x_1(t), x_2(t))$ is $(\theta(t), \dot{\theta}(t))$, where θ is the angle that the pole makes with vertical line and the control input u is the applied force to the cart. For dynamic parameters, we set $m_c = 1$ kg, $m = 0.1$ kg, and $l = 0.5$ m. For control and learning parameters, we set $\bar{\mathbf{k}} = 3$, $\rho = 2.5$, $k_g = 1$, and $\alpha(t) = |\dot{x}^j(t)| + |x^j(t)|$. For learning and adaptation gains, we set $\beta_1 = 5$, $\beta_h = 4.75$, $\beta_2 = 0.5$, and $\beta_3 = 0.2$. Finally, for initial states, we set $\mathbf{x}(0) = (\pi/30, 0)$, and for sampling interval, we use 0.05 sec. In this simulation, $x_i(t)$ is limited to $|x_i(t)| < 0.3$ and the membership functions are set to $\mu_{F_i^1} = \exp[-((x_i + 0.3)/(\pi/24))^2]$, $\mu_{F_i^2} = \exp[-((x_i + 0.1)/(\pi/12))^2]$, $\mu_{F_i^3} = \exp[-(x_i/(\pi/6))^2]$, $\mu_{F_i^4} = \exp[-((x_i - 0.1)/(\pi/12))^2]$, and $\mu_{F_i^5} = \exp[-((x_i - 0.3)/(\pi/24))^2]$, for $i = 1, 2$. To obtain a positive effect of adaptive fuzzy compensation block, β_2 of $h^j(t)$ should be greater than β_3 of $u_c^j(t)$. In fact, since $\theta(0)$ of the fuzzy compensator are set to zero in this simulation, a large value of β_3 may drive $u_c^j(t)$ as a large disturbance for the system. If partial information on the system is available, it can be reflected on the initial parameter $\theta(0)$ to compensate for the nonlinear term. Figures 1 and 3 show the trajectories of $x_1(t)$ and $x_2(t)$ when the fuzzy compensator is not used whereas Figures 2 and 4 show the trajectories of $x_1(t)$ and $x_2(t)$ when the fuzzy compensator is used in the controller design. Figure 5 shows the summed control input magnitude $(\sum_{k=1}^{N} |u(\Delta t - k)|)$, where $N = 300$ and $\Delta t = 0.05$ sec, which covers the time span $0 \le t \le 15$(sec). The simulation

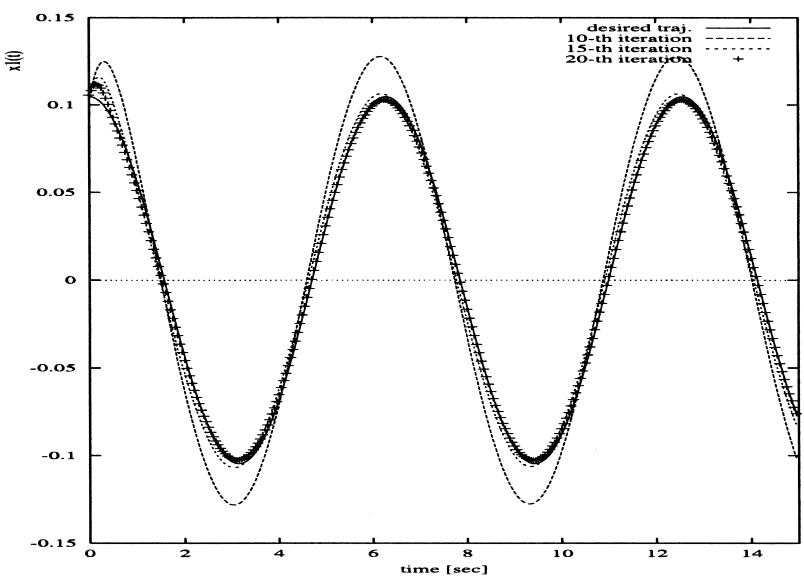

Figure 1. Trajectory of $x_1(t)$ without fuzzy compensator.

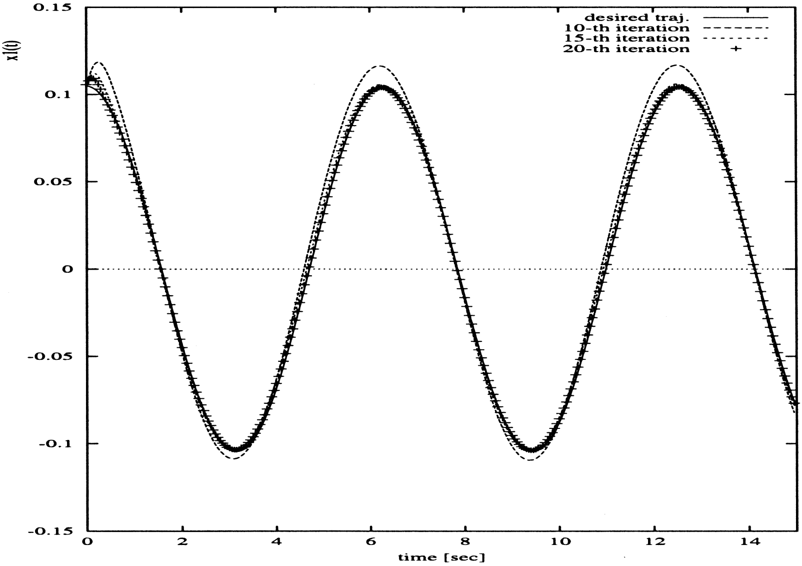

Figure 2. Trajectory of $x_1(t)$ with fuzzy compensator.

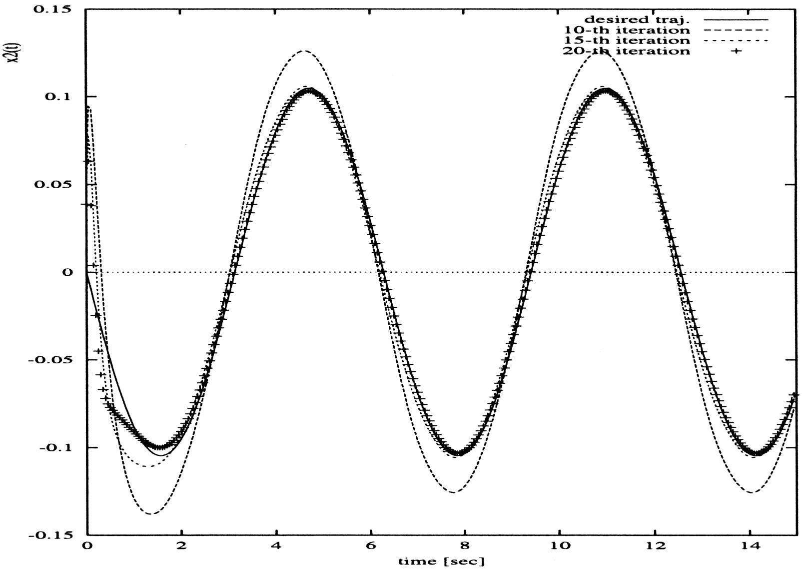

Figure 3. Trajectory of $x_2(t)$ without fuzzy compensator.

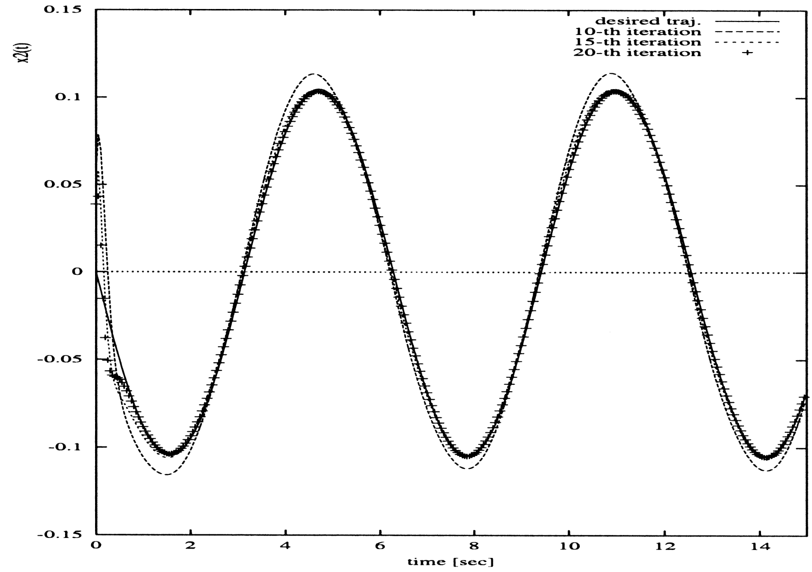

Figure 4. Trajectory of $x_2(t)$ with fuzzy compensator.

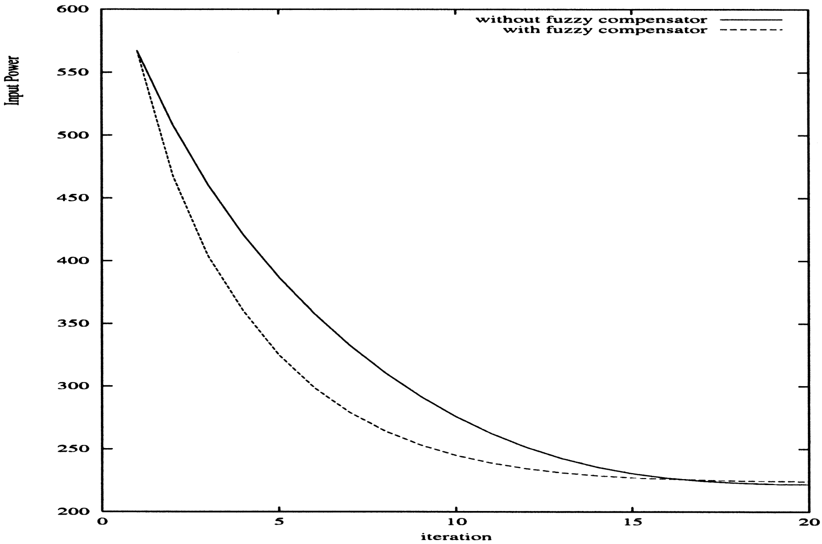

Figure 5. Summed input power for every iteration.

results show that the learning controller with the fuzzy compensation term enhances the tracking performance without increasing the corresponding control input magnitude.

7. CONCLUSIONS

An adaptive learning control scheme has been presented for a class of nonlinear dynamic systems. The controller presented extends the result of Park et al.[8] to a class of general nonlinear systems by using the fuzzy approximation technique for nonlinear compensation. The controller presented consists of three control units: feedback control unit, nonlinear compensation unit, and learning control unit. The feedback controller provides stability of the system and keeps its state errors within uniform bounds. The nonlinear compensation term reduces the control load from the feedback control term and keeps the feedback gains reasonably small. The learning controller achieves precise tracking without using any output derivative terms which are vulnerable to noise. The adaptive nonlinear compensation term is determined by using the fuzzy approximation technique for a class of nonlinear systems. Simulation results show that the fuzzy compensator enhances tracking performance without increasing input power. We can see that the control input power can be reduced with a properly chosen fuzzy compensator from Figure 5. The controllers developed are rather complex and normally require much computation power and memory for the generation of real-time control input. Fortunately, CPU power and memory have become less of a problem these days because with the aid of technology, fast processors are available at a low price.

APPENDIX A

Proof of Lemma 1. Note that

$$|w_g^j - w_{gd}| = |(g^j - \hat{g}^{*j}) - (g_d - \hat{g}_d^*)| \leq |g^j - g_d| + |\hat{g}^{*j} - \hat{g}_d^*| \qquad \text{and}$$

$$|w_f^j - w_{fd}| = |(f^j - \hat{f}^{*j}) - (f_d - \hat{f}_d^*)| \leq |f^j - f_d| + |\hat{f}^{*j} - \hat{f}_d^*|$$

Then using Assumption 1 and the mean-value theorem, we have

$$|w_g^j - w_{gd}| \leq \left(l_g + \left\|\frac{\partial \hat{g}^*}{\partial \mathbf{x}}\right\|\right) \cdot |\mathbf{x}^j - \mathbf{x}_d| \leq \left(l_g + \left\|\frac{\partial \zeta_g^T(\mathbf{x})}{\partial \mathbf{x}}\right\| \cdot \theta_g^b\right) \cdot |\mathbf{x}^j - \mathbf{x}_d| \quad (19)$$

and

$$|w_f^j - w_{fd}| \leq \left(l_f + \left\|\frac{\partial \hat{f}^*}{\partial \mathbf{x}}\right\|\right) \cdot |\mathbf{x}^j - \mathbf{x}_d| \leq \left(l_f + \left\|\frac{\partial \zeta_f^T(\mathbf{x})}{\partial \mathbf{x}}\right\| \cdot \theta_f^b\right) \cdot |\mathbf{x}^j - \mathbf{x}_d| \quad (20)$$

Then, it follows that

$$
\begin{aligned}
|w_e^j| &= \left| \left(w_g^j - w_{gd} \right) x_d^{(n)} + w_f^j - w_{fd} - g^j \bar{\mathbf{k}}^T \dot{\bar{\mathbf{e}}}^j \right| \\
&\leq \left(\|x_d^{(n)}\| r_g + r_f + g^U |\bar{\mathbf{k}}| \right) \cdot |\mathbf{x}^j - \mathbf{x}_d|
\end{aligned} \tag{21}
$$

where

$$
r_g = \left(l_g + \left\| \frac{\partial \zeta_g{}^T(\mathbf{x})}{\partial \mathbf{x}} \right\| \cdot \theta_g^b \right) \qquad \text{and} \qquad r_f = \left(l_f + \left\| \frac{\partial \zeta_f{}^T(\mathbf{x})}{\partial \mathbf{x}} \right\| \cdot \theta_f^b \right)
$$

Note here that the values of $\|(\partial \zeta_g{}^T(\mathbf{x}))/\partial \mathbf{x}\|$ and $\|(\partial \zeta_f{}^T(\mathbf{x}))/\partial \mathbf{x}\|$ can be calculated off-line once the fuzzy membership functions are determined. Hence, $l_w = \|x_d^{(n)}\| r_g + r_f + g^U |\bar{\mathbf{k}}|$. ■

APPENDIX B

Proof of Lemma 2. The lemma is proved by mathematical induction. For $n = 1$, it holds that

$$
\begin{aligned}
\int_0^t e_1^2(\tau)\, d\tau &= \int_0^t \left(\frac{de(\tau)}{d\tau} + ae(\tau) \right)^2 d\tau \\
&= \int_0^t \left(\left(\frac{de(\tau)}{d\tau} \right)^2 + a^2 e^2(\tau) \right) d\tau + ae^2(t)
\end{aligned}
$$

which is (10). For $n = m$, suppose that

$$
\begin{aligned}
\int_0^t e_m^2(\tau)\, d\tau &= \int_0^t \sum_{i=0}^{m} {}_mC_i \left(\frac{d^i e(\tau)}{d\tau^i} \right)^2 a^{2(m-i)}\, d\tau \\
&\quad + a \sum_{k=0}^{m-1} \sum_{i=0}^{k} {}_kC_i \left(\frac{d^i e_{m-k-1}(t)}{d\tau^i} \right)^2 a^{2(k-i)}
\end{aligned}
$$

Then, for $n = m + 1$, $e_{m+1} = ((d/dt) + a)^m ((d/dt) + a) e(t) = ((d/dt) + a)^m$ $e_1(t)$ such that

$$
\begin{aligned}
&\int_0^t e_{m+1}^2(\tau)\, d\tau \\
&= \int_0^t \sum_{i=0}^{m} {}_mC_i \left(\frac{d^i e_1(\tau)}{d\tau^i} \right)^2 a^{2(m-i)}\, d\tau + a \sum_{k=0}^{m-1} \sum_{i=0}^{k} {}_kC_i \left(\frac{d^i e_{m-k}(t)}{dt^i} \right)^2 a^{2(k-i)} \\
&= \int_0^t \sum_{i=0}^{m} {}_mC_i \left(\left(\frac{d^{i+1} e(\tau)}{d\tau^{i+1}} \right)^2 a^{2(m-i)} + \left(\frac{d^i e(\tau)}{d\tau^i} \right)^2 a^{2(m+1-i)} \right) d\tau \\
&\quad + a \sum_{i=0}^{m} {}_mC_i \left(\frac{d^i e(t)}{dt^i} \right)^2 a^{2(m-i)} + a \sum_{k=0}^{m-1} \sum_{i=0}^{k} {}_kC_i \left(\frac{d^i e_{m-k}(t)}{dt^i} \right)^2 a^{2(k-i)}
\end{aligned}
$$

$$= \int_0^t \left(\left(\frac{d^{m+1}e(\tau)}{d\tau^{m+1}} \right)^2 + \sum_{i=1}^{m} ({}_mC_{i-1} + {}_mC_i) \left(\frac{d^ie(\tau)}{d\tau^i} \right)^2 a^{2(m+1-i)} \right.$$

$$\left. + e^2(t) a^{2(m+1)} \right) d\tau$$

$$+ a \sum_{k=0}^{m} \sum_{i=0}^{k} {}_kC_i \left(\frac{d^i e_{m-k}(t)}{dt^i} \right)^2 a^{2(k-i)}$$

$$= \int_0^t \sum_{i=0}^{m+1} {}_{m+1}C_i \left(\frac{d^ie(\tau)}{d\tau^i} \right)^2 a^{2(m+1-i)} \, d\tau$$

$$+ a \sum_{k=0}^{m} \sum_{i=0}^{k} {}_kC_i \left(\frac{d^i e_{m-k}(t)}{dt^i} \right)^2 a^{2(k-i)}$$

Since (10) is satisfied for $n = m + 1$, it follows that (10) holds for all integer $n \geq 1$. ■

APPENDIX C

Proof of Theorem 1. First, we show that $|z^j|$ is bounded. Let $\tilde{\bar{u}}^j$ be $\bar{h}^j - u_d$. Then, $|\tilde{\bar{u}}^j| \geq |\tilde{u}^j|$ and $v^{j+1}(t) - v^j(t) \leq \bar{v}^{j+1}(t) - \bar{v}^j(t)$, where $\bar{v}^j(t) = \int_0^t \tilde{\bar{u}}^{j^2} d\tau$. Let $\Delta\tilde{u}^j$ be $\tilde{\bar{u}}^{j+1} - \tilde{u}^j$. Then, $\Delta\tilde{u}^j = \tilde{\bar{u}}^{j+1} - \tilde{u}^j = \bar{h}^{j+1} - h^j = -\beta_2 z^j$ and

$$v^{j+1}(t) - v^j(t)$$

$$\leq \bar{v}^{j+1}(t) - v^j(t)$$

$$= \int_0^t \left(\tilde{\bar{u}}^{j+1^2} - \tilde{u}^{j^2} \right) d\tau$$

$$= \int_0^t \left(\Delta\tilde{u}^{j^2} + 2\Delta\tilde{u}^j\tilde{u}^j \right) d\tau$$

$$= \int_0^t \left(\beta_2^2 z^{j^2} - 2\beta_2 z^j \left(g^j \dot{z}^j + \beta_1 z^j + k_g \alpha_g(t) z^j + w_e^j \right) d\tau \right.$$

$$= \int_0^t \left(\left(\beta_2^2 - 2\beta_1 \beta_2 \right) z^{j^2} - 2\beta_2 z^j g^j \dot{z}^j - 2\beta_2 k_g \alpha_g(t) z^{j^2} - 2\beta_2 w_e^j z^j \right) d\tau$$

$$\leq -\beta_2 z^{j^2} g^j$$

$$+ \int_0^t \left(\beta_2(1 - 2k_g) \alpha_g(\tau) z^{j^2} + \left(\beta_2^2 - 2\beta_1 \beta_2 \right) z^{j^2} + 2\beta_2 |w_e^j| |z^j| \right) d\tau$$

(integration by parts)

$$\leq -\beta_2 z^{j^2} g^j$$

$$-2\beta_2 \int_0^t \left(\beta_h z^{j^2} - l_w \sum_{k=0}^{n-1} \sum_{i=0}^{n-1} {}_{n-1}C_i \left| \frac{d^i e^j(t)}{dt^i} \right| \left| \frac{d^k e^j(t)}{dt^k} \right| a^{n-1-i} \right) d\tau$$

$$\leq -\beta_2 z^{j^2} g^j - 2\beta_2 \int_0^t \left(\beta_h \sum_{i=0}^{n-1} {}_{n-1}C_i \left(\frac{d^i e^j(t)}{dt^i} \right)^2 a^{2(n-1-i)} \right.$$

$$\left. - l_w \sum_{k=0}^{n-1} \sum_{i=0}^{n-1} {}_{n-1}C_i \left| \frac{d^i e^j(t)}{dt^i} \right| \left| \frac{d^k e^j(t)}{dt^k} \right| a^{n-1-i} \right) d\tau \qquad \text{(by Lemma 2)}$$

$$= -\beta_2 z^{j^2} g^j$$

$$-2\beta_2 \int_0^t \left(\beta_h \sum_{i=0}^{n-1} k_i' \left(\frac{d^i e^j(t)}{dt^i} \right)^2 - l_w \sum_{k=0}^{n-1} \sum_{i=0}^{n-1} k_i \left| \frac{d^i e^j(t)}{dt^i} \right| \left| \frac{d^k e^j(t)}{dt^k} \right| \right) d\tau$$

$$\leq -\beta_2 z^{j^2} g^j - 2\beta_2 \int_0^t \left(\mathbf{e}^{j^T} Q \mathbf{e}^j \right) d\tau \qquad \text{(by Lemma 3)}$$

$$\leq -\beta_2 z^{j^2} g^j$$

Thus, $v^1 \geq v^j - v^{j+1} \geq \beta_2 g^{j^2} \geq g_L \beta_2 z^{j^2}$ for all j ≥ 1, which proves that $|z^j|$ is bounded. Next, since v^j is positive definite and monotonically decreasing, it converges to some positive function v from Ref. 19, which is (i). From the convergence of v^j, it follows that there exists a positive integer N for all $\sigma > 0$ such that $|v^{j+1}(t) - v^j(t)| \leq \sigma$ for all $j \geq N$. If we choose $\sigma = \beta_2 g_L \epsilon^2$ given any $\epsilon \leq 0$, then we have

$$\beta_2 g_L |z^j(t)|^Z \leq v^j(t) - v^{j+1}(t) \leq \beta_2 g_L \epsilon^2 \qquad \text{for all } j \geq N \tag{22}$$

Hence, it follows that $|z^j(t)| \leq \epsilon$ for any $\epsilon > 0$, from which (ii) follows. ■

APPENDIX D

PROOF OF THEOREM 2. First we prove that $|z^j|$ is bounded. Since $|\tilde{\bar{u}}^j| \geq |\tilde{u}^j|$ and $|\tilde{\bar{\theta}}^j| \geq |\tilde{\theta}^j|$, we have $v_a^{j+1} - v_a^j \leq \bar{v}_a^{j+1} - v_a^j$, where $\bar{v}_a^j(t) = \int_0^t (\tilde{\bar{u}}^{j^2}(\tau) + \rho \tilde{\bar{\theta}}^{j^T}(\tau) \tilde{\bar{\theta}}^j(\tau))\, d\tau$ and $\tilde{\bar{\theta}}^j = \theta^* - \hat{\bar{\theta}}^j$. Let $\Delta\tilde{\theta}^j$ be $\tilde{\bar{\theta}}^{j+1} - \tilde{\theta}^j$. Then $\Delta\tilde{\theta}^j = \tilde{\bar{\theta}}^{j+1} - \tilde{\theta}^j = \hat{\theta}^j - \hat{\bar{\theta}}^{j+1} = -\beta_3 Y_s^j z^j$ and

$$v_a^{j+1}(t) - v_a^j(t)$$

$$\leq \bar{v}_a^{j+1}(t) - v_a^j(t)$$

$$= \int_0^t \left(\tilde{\bar{u}}^{j+1^2} - \tilde{u}^{j^2} + \rho \left(\tilde{\bar{\theta}}^{j+1^T} \tilde{\bar{\theta}}^{j+1} - \tilde{\theta}^{j^T} \tilde{\theta}^j \right) \right) d\tau$$

$$= \int_0^t \Big(\Delta\tilde{u}^{j^2} + 2\Delta\tilde{u}^j\tilde{u}^j + \rho\big(\Delta\tilde{\theta}^{j^T}\Delta\tilde{\theta}^j + 2\Delta\tilde{\theta}^{j^T}\tilde{\theta}^j\big)\Big)\, d\tau$$

$$= \int_0^t \Big(\beta_2^2 z^{j^2} - 2\beta_2 z^j \Big(g^j\dot{z}^j + \beta_1 z^j + k_g\alpha_g(t)z^j + w_e^j + Y_s^j Y_s^{j^T} z^j - Y_s^j\tilde{\theta}^j \Big)$$

$$+ \rho\Big(\beta_3^2 z^{j^2} Y_s^j Y_s^{j^T} - 2\beta_3 z^j Y_s^j \tilde{\theta}^j \Big)\Big)\, d\tau$$

$$= \int_0^t \Big(\beta_2^2 z^{j^2} - 2\beta_2 z^j \Big(g^j\dot{z}^j + \beta_1 z^j + k_g\alpha_g(t)z^j + w_e^j \Big)\, d\tau$$

$$+ \int_0^t \Big((-2\beta_2 + \rho\beta_3^2) z^{j^2} Y_s^j Y_s^{j^T} + 2(\beta_2 - \rho\beta_3) z^j Y_s^j \tilde{\theta}^j \Big)\, d\tau$$

$$\le -\beta_2 z^{j^2} g^j$$

The final step can be obtained by following the same procedure as that in the proof of Theorem 1. Thus, $v_a^1 \ge v_a^j - v_a^{j+1} \le \beta_2 g_L z^{j^2}$ for all $j \ge 1$, which proves that $|z^j|$ is bounded. Next, since v_a^j is positive definite and monotonically decreasing, v_a^j converges to some positive function v_a from Ref. 19 and (ii) follows by using the same argument as that in the proof of Theorem 1. ■

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