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PAPER

Global Exponential Stability of FAST TCP with Heterogeneous Time-Varying Delays*

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SUMMARY We address the stability property of the FAST TCP congestion control algorithm. Based on a continuous-time dynamic model of the FAST TCP network, we establish that FAST TCP in itself is globally exponentially stable without any specific conditions on the congestion control parameter or the update gain. Simulation results demonstrate the validity of the global exponential stability of FAST TCP.

key words: *internet congestion control, FAST TCP, nonlinear time delay systems, global exponential stability*

1. Introduction

Internet congestion control is a distributed feedback control algorithm to allocate network capacity among competing users, and has been commonly implemented by TCP Reno and its variants, which control their congestion window based on the well-known *additive increase multiplicative decrease* (AIMD) mechanism [1], [2]. It has been, however, shown that these algorithms are not scalable as the bandwidth-delay product of the network becomes larger because additive increasing is too slow and multiplicative decreasing is too severe in the large bandwidth-delay product network [3]–[6].

In order to cope with this problem, several congestion control algorithms have been proposed for high speed networks with large bandwidth-delay products: HSTCP [7], STCP [8], FAST TCP [5], and BIC TCP [9]. Among them, FAST TCP has a feature that the queuing delay is used as a congestion measure. While the packet loss that is used as a congestion measure in TCP Reno, HSTCP and STCP has only binary information about the congestion, the queuing delay indicates a level of congestion, which means how far the current state is from the equilibrium. Accordingly, FAST TCP shows high utilization of networks and scalability with the increase of bandwidth-delay products [6], but nonetheless these advantages of FAST TCP can be secured by the stability of FAST TCP in which the network states stay near the equilibrium.

Several research efforts have been devoted to the analysis of the stability property of FAST TCP. The local stability of FAST TCP is investigated in [6], [10]–[12] by linearizing the nonlinear dynamic model of FAST TCP around the equilibrium points.

On the other hand, in order to ensure that FAST TCP converges to its equilibrium from any initial state, the global stability of FAST TCP is important and should be rigorously considered. In [13], [14], a global asymptotic stability condition was established for FAST TCP in a single-link single-source network in the presence of network feedback delay. In [15], a continuous-time dynamic model was proposed for FAST TCP in a single-link multi-source network, but the global exponential stability of each source was not clearly proved and the time-varying property of RTT(round trip time) was not considered in the analysis. In [16], a global asymptotic stability condition was established for FAST TCP in a single-link multi-source network in the presence of network feedback delay. All of the stability conditions established in [13], [14], [16] require that the congestion control parameter α should be greater than a positive constant that is dependent on the network environment parameters, or the update gain γ should be less than a positive constant that is dependent on the network parameters.

In this paper, we clearly prove the global exponential stability of a single-link multi-source network with the FAST TCP sources, based on the dynamic model of FAST TCP that was proposed in [14], [15]. We establish that FAST TCP under congestion in itself is globally exponentially stable even in the presence of time-varying network feedback delay without any specific conditions on the congestion control parameter α or the update gain γ.

This paper is organized as follows. Section 2 presents a dynamic model for the single-link multi-source network with the FAST TCP sources. Section 3 analyzes the nonnegative property of the FAST TCP model. Section 4 shows the global exponential stability of FAST TCP. Section 5 provides the simulation results. Section 6 makes conclusions.

2. Network Model

We consider a single-link multi-source network, where a single communication link is shared by N FAST TCP sources as depicted in Fig. 1. The link has a finite transmission capacity c and is assumed to have infinite buffering

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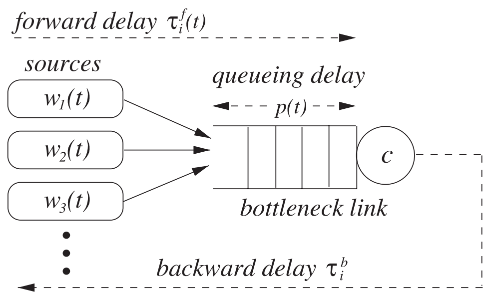


Fig. 1 Network model.

storage. Each source is indexed by $i (1 \le i \le N)$. The queuing delay $p(t)$ is associated with the link and the congestion window $w_i(t)$ is with the source i. We assume that the source i observes the queuing delay $q_i(t)$ as a feedback signal in its path:

$$q_i(t) := p(t - \tau_i^b), \tag{1}$$

where τ_i^b denotes the backward delay in the feedback path from link to source i, and the link observes the aggregated congestion window

$$y(t) := \sum_i w_i(t - \tau_i^f), \tag{2}$$

where τ_i^f denotes the forward delay from source i to link. Then, the network feedback delay of source i amounts to $\tau_i^f + \tau_i^b$ in the network model.

On the other hand, in TCP, the round trip time (RTT) $T_i(t)$ of source i is defined as the time delay between the first bit of a packet being transmitted by the sender and the last bit of its associated acknowledgment being received. The RTT of source i is defined as $T_i(t) := d_i + q_i(t)$, where d_i is the constant round trip propagation delay. By comparing the definition of RTT with the network feedback delay of the network model, we obtain that $T_i(t) = \tau_i^f + \tau_i^b$, which implies that the network feedback delay $\tau_i^f + \tau_i^b$ is time-varying.

For the feasibility of analysis, we assume that the forward delay $\tau_i^f(t)$ is time-varying and the backward delay τ_i^b is a constant, that is, $T_i(t) = \tau_i^f(t) + \tau_i^b$. This assumption is reasonable because the time-varying part $q_i(t)$ of $T_i(t)$ is totally included in the forward delay $\tau_i^f(t)$ as shown in Fig. 1. In this framework of the network model, we adopt a continuous-time model of the FAST TCP network under congestion from [14], [15].

In [14], [15], the dynamic model for the FAST TCP source i under congestion was represented by the following time delay differential equation:

$$\dot{w}_i(t) = \gamma\left(-w_i(t) + \frac{d_i}{d_i + p(t - \tau_i^b)} w_i(t - T_i(t)) + \alpha_i\right) \tag{3}$$

with the initial condition

$$w_i(t_0 + \theta) = \phi_i(\theta) \ge 0, \quad \theta \in [-T^M, 0], \tag{4}$$

where t_0 is the initial time, $\gamma \in (0, 1]$ is the update gain, $\alpha_i > 0$ is the congestion control parameter, ϕ_i is a continuous function such that $\phi_i : [-T^M, 0] \mapsto \mathbb{R}_+$, $\mathbb{R}_+$ is the set of non-negative real numbers, and $T^M := \max_{i, t \ge t_0} T_i(t)$. In order to reflect that the congestion window is always non-negative in real networks, we set the initial condition of $w_i(t)$ as non-negative values in (4), which will be justified in Lemma 1 in Sect. 3.

On the other hand, in [14], [15], the queuing delay model at the link under congestion was represented by

$$\sum_j \frac{w_j(t - \tau_j^f(t))}{d_j + p(t)} = c, \tag{5}$$

where $p(t) > 0$ and $\sum_j \frac{w_j(t - \tau_j^f(t))}{d_j} > c$ because the network is under congestion. The whole closed-loop system consists of the congestion window adjustment rule of each source described by (3) and the queuing delay described by (5).

As shown in (3) and (5), the whole closed-loop system has high nonlinearity along with heterogeneous state-dependent time-varying delays because $T_i(t) = d_i + p(t - \tau_i^b) = \tau_i^f(t) + \tau_i^b$. It is remarkable that we maintain the state-dependent time-varying property of the network feedback delay $T_i(t)$ in the subsequent analysis, while the network feedback delay was assumed to be constant in [10], [12], [13], [17].

3. Non-negativity

In this section, we analyze the non-negative property of $w_i(t)$ with respect to time t. We show in the following lemma that $w_i(t)$ is bounded below by zero.

Lemma 1: The congestion window $w_i(t)$ described by (3) with the initial condition (4) is bounded below as $w_i(t) \ge 0$ for all $t \ge t_0$.

Proof: The solution of (3) is given by the variation-of-constants formula [18, p.12] as

$$w_i(t) = e^{-\gamma(t - t_0)} w_i(t_0) + \gamma e^{-\gamma t} \int_{t_0}^{t} e^{\gamma s} \left(\frac{d_i}{d_i + p(s - \tau_i^b)} w_i(s - T_i(s)) + \alpha_i\right) ds. \tag{6}$$

Since $w_i(t_0 + \theta) \ge 0$ for all $-T^M \le \theta \le 0$ from (4), it holds that $w_i(s - T_i(s)) \ge 0$ for all $t_0 \le s \le t_0 + T_i(s_1)$, where s_1 satisfies the equation $(s_1 - T_i(s_1) = t_0)$ and $T_i(s_1) \ge d_i > 0$ by the definition of RTT as $T_i(t) := d_i + q_i(t)$. Moreover, since $p(s - \tau_i^b) \ge 0$ by the definition of (5) and $\alpha_i > 0$ by the definition of α_i in (3), it is obvious that $\left(\frac{d_i}{d_i + p(s - \tau_i^b)} w_i(s - T_i(s)) + \alpha_i\right) \ge 0$ for all $t_0 \le s \le t_0 + T_i(s_1)$ in (6). Hence, we obtain from (6) that $w_i(t) \ge 0$ for all $t_0 \le t \le t_0 + T_i(s_1)$, which also guarantees that $w_i(s - T_i(s)) \ge 0$ for all $s_1 \le s \le s_2$, where s_2 satisfies the equation

$(s_2 - T_i(s_2) = t_0 + T_i(s_1))$ and $T_i(s_2) \geq d_i > 0$. That is, we have $w_i(s - T_i(s)) \geq 0$ for all $t_0 + T_i(s_1) \leq s \leq t_0 + T_i(s_1) + T_i(s_2)$, and we come to obtain from (6) that $w_i(t) \geq 0$ for all $t_0 + T_i(s_1) \leq t \leq t_0 + T_i(s_1) + T_i(s_2)$. Recursively, extending this procedure for all $t \geq t_0$, we achieve that $w_i(t) \geq 0$ for all $t \geq t_0$. □

Note that Lemma 1 justifies selecting the initial condition of $w_i(t)$ in (4) as non-negative values, and it is verified that the congestion window model (3) adequately describes the non-negative property of congestion windows in real networks.

4. Global Exponential Stability

For the facilitation of analysis, we introduce a delayed congestion window $z_i(t)$ and a delayed queuing delay $p_M(t)$ such that

$$z_i(t) := w_i(t + \tau_i^b - \tau_M^b) \text{ and } p_M(t) := p(t - \tau_M^b), \tag{7}$$

where $\tau_M^b := \max_i \tau_i^b$. Then, the congestion window model (3) under congestion is rewritten in terms of $z_i(t)$ and $p_M(t)$ as

$$\dot{z}_i(t) = \gamma\left(-z_i(t) + \frac{d_i}{d_i + p_M(t)} z_i(t - T_i(t)) + \alpha_i\right) \tag{8}$$

with the initial condition $z_i(t_0 + \theta) = \zeta_i(\theta) \geq 0$ for $\theta \in [-T^M, 0]$, where ζ_i is a continuous function such that $\zeta_i : [-T^M, 0] \mapsto \mathbb{R}_+$. Similarly, the queuing delay model (5) is rewritten in terms of $z_i(t)$ and $p_M(t)$ as

$$\sum_j \frac{z_j(t - T_j(t))}{d_j + p_M(t)} = c, \tag{9}$$

where $p_M(t) > 0$ and $\sum_j \frac{z_j(t-T_j(t))}{d_j} > c$ because the network is under congestion. Since $0 \leq \tau_M^b - \tau_i^b < \infty$ and $0 \leq \tau_M^b < \infty$ in (7), it follows that the exponential stability of $w_i(t)$ and $p(t)$ is equivalent to that of $z_i(t)$ and $p_M(t)$. Hence, we will show the global exponential stability of $z_i(t)$ and $p_M(t)$ in this section, which establishes the global exponential stability of $w_i(t)$ and $p(t)$.

In order to show the global exponential stability of (8) and (9), we need the following lemma that is a modified version of the lemma in [19].

Lemma 2: Let $g(t)$ be a continuous function with $g(t) \geq 0$ for all $t \geq t_0 - \tau$. Let

$$\dot{g}(t) \leq -\beta_1 g(t) + \beta_2 \sup_{t-\tau \leq s \leq t} g(s) \text{ for } t \geq t_0, \tag{10}$$

where β_1 and β_2 are positive constants. If $0 < \beta_2 < \beta_1$ then there exist a constant $\eta > 0$ such that

$$g(t) \leq |g_{t_0}| e^{-\eta(t-t_0)} \text{ for } t \geq t_0, \tag{11}$$

where $|g_{t_0}| := \sup_{t_0-\tau \leq s \leq t_0} g(s)$.

Proof: In the same way as in the proof of the lemma in [19], this lemma can be easily proved. □

Next, we derive error equations for the delayed congestion window $z_i(t)$ and the delayed queuing delay $q_M(t)$. For this purpose, we obtain the unique equilibrium points z_i^* and p_M^* of $z_i(t)$ and $q_M(t)$ in (8) and (9) by solving the following algebraic equations:

$$-z_i^* + \frac{d_i}{d_i + p_M^*} z_i^* + \alpha_i = 0 \text{ for } 1 \leq i \leq N$$
$$\sum_{j=1}^N \frac{z_j^*}{d_j + p_M^*} = c,$$

where the equilibrium points are computed as

$$z_i^* = \alpha_i + \frac{\alpha_i}{\sum_j \alpha_j} c d_i \text{ for } 1 \leq i \leq N \text{ and } p_M^* = \frac{\sum_j \alpha_j}{c}. \tag{12}$$

Using these equilibrium points z_i^* and p_M^*, we define the delayed congestion window error as $\tilde{z}_i(t) := z_i(t) - z_i^*$ and delayed queuing delay error as $\tilde{p}_M(t) := p_M(t) - p_M^*$. Based on (8) and (9), we derive the error equations for the delayed congestion window and the delayed queuing delay as

$$\frac{\dot{\tilde{z}}_i(t)}{\alpha_i d_i} = \gamma\left(-\frac{1}{\alpha_i d_i}\tilde{z}_i(t) + \frac{1}{\alpha_i(d_i + p_M(t))}\tilde{z}_i(t - T_i(t)) - \frac{c}{\sum_j \alpha_j}\frac{1}{d_i + p_M(t)}\tilde{p}_M(t)\right) \tag{13}$$

$$\tilde{p}_M(t) = \left(\frac{c}{\sum_j \alpha_j}\sum_j \frac{\alpha_j}{d_j + p_M(t)}\right)^{-1} \sum_j \frac{\tilde{z}_j(t - T_j(t))}{d_j + p_M(t)}. \tag{14}$$

Substituting (14) for $\tilde{p}_M(t)$ in (13) yields

$$\frac{\dot{\tilde{z}}_i(t)}{\alpha_i d_i} = \gamma\left(-\frac{1}{\alpha_i d_i}\tilde{z}_i(t) + \frac{1}{\alpha_i d_{ip}}\tilde{z}_i(t - T_i(t)) - \frac{1}{d_{ip}}\left(\sum_j \frac{\alpha_j}{d_{jp}}\right)^{-1}\sum_j \frac{\tilde{z}_j(t - T_j(t))}{d_{jp}}\right), \tag{15}$$

where $d_{ip} := d_i + p_M(t)$ for all $1 \leq i \leq N$. Defining a vector $\tilde{z}$ such that $\tilde{z} := [\tilde{z}_1\ \tilde{z}_2\ \cdots\ \tilde{z}_N]^T \in \mathbb{R}^N$, (15) is expressed in a vector-matrix form:

$$\frac{1}{\gamma}A\dot{\tilde{z}}(t) = -A\tilde{z}(t) + (B(t) - C(t))\,\tilde{z}(t - T_i(t)), \tag{16}$$

where

$$A := \mathrm{diag}\left(\frac{1}{\alpha_1 d_1}, \frac{1}{\alpha_2 d_2}, \cdots, \frac{1}{\alpha_N d_N}\right), \tag{17}$$

$$B(t) := \mathrm{diag}\left(\frac{1}{\alpha_1 d_{1p}}, \frac{1}{\alpha_2 d_{2p}}, \cdots, \frac{1}{\alpha_N d_{Np}}\right), \tag{18}$$

$$C(t) := \frac{1}{\left(\sum_j \frac{\alpha_j}{d_{jp}}\right)}\begin{bmatrix} \frac{1}{d_{1p}^2} & \frac{1}{d_{1p}d_{2p}} & \cdots & \frac{1}{d_{1p}d_{Np}} \\ \frac{1}{d_{2p}d_{1p}} & \frac{1}{d_{2p}^2} & \cdots & \frac{1}{d_{2p}d_{Np}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{d_{Np}d_{1p}} & \frac{1}{d_{Np}d_{2p}} & \cdots & \frac{1}{d_{Np}^2} \end{bmatrix}, \tag{19}$$

diag $(a_1, a_2, \cdots, a_N)$ is an $N \times N$ diagonal matrix with a_i as its i-th diagonal element, and A, $B(t)$, $C(t) \in \mathbb{R}^{N\times N}$. Note that A and $B(t)$ are positive definite, and $C(t)$ is symmetric. The following lemmas are required to show the global exponential stability of (16).

Lemma 3: $C(t)$ in (19) is positive semidefinite; $(B(t) - C(t))$ in (16) is positive semidefinite.

Proof: This lemma can be easily proved by showing that $x^T C(t)x \geq 0$ and $x^T (B(t) - C(t))\, x \geq 0$ for all $x = [x_1\ x_2\ \cdots\ x_N]^T \in \mathbb{R}^N$. See Appendix for the detailed proof to show $x^T (B(t) - C(t))\, x \geq 0$. □

Lemma 4: For diagonal positive definite matrices A and $B(t)$ defined in (17) and (18), there exists a diagonal positive definite matrix $D \in \mathbb{R}^{N\times N}$ such that both $(A - D)$ and $(D - B(t))$ are positive definite. Furthermore, there exists a constant $0 < \delta < 1$ such that $x^T Dx \leq \delta x^T Ax$ for all $x \in \mathbb{R}^N$.

Proof: We prove this lemma by constructing the diagonal positive definite matrix $D \in \mathbb{R}^{N\times N}$. Under congestion, we can find a constant ϵ such that $p_M(t) \geq \epsilon$, and define a diagonal matrix B^M as

$$B^M := \text{diag}\left(\frac{1}{\alpha_1(d_1+\epsilon)}, \frac{1}{\alpha_2(d_2+\epsilon)}, \cdots, \frac{1}{\alpha_N(d_N+\epsilon)} \right), \tag{20}$$

which implies that $x^T Bx \leq x^T B^M x$. Choosing a diagonal matrix D as

$$D := \text{diag}\left(\frac{1}{\alpha_1(d_1+\epsilon/2)}, \frac{1}{\alpha_2(d_2+\epsilon/2)}, \cdots, \frac{1}{\alpha_N(d_N+\epsilon/2)} \right), \tag{21}$$

it is obvious that $(A - D)$ and $(D - B(t))$ are positive definite by comparing diagonal elements of D with those of A in (17), and those of B^M in (20). Namely, $\frac{1}{\alpha_i d_i} > \frac{1}{\alpha_i(d_i+\epsilon/2)}$ and $\frac{1}{\alpha_i(d_i+\epsilon/2)} > \frac{1}{\alpha_i(d_i+\epsilon)}$ for all $1 \leq i \leq N$. Furthermore, we derive from A in (17) and D in (21) that

$$\begin{aligned} x^T Dx &= x^T DA^{-1}Ax \\ &\leq \max_i \left(\frac{\alpha_i d_i}{\alpha_i (d_i + \epsilon/2)} \right) x^T Ax \\ &= \delta x^T Ax, \end{aligned} \tag{22}$$

where $\delta := \max_i \left(\frac{\alpha_i d_i}{\alpha_i(d_i+\epsilon/2)} \right) < 1$. □

At last, we establish the global exponential stability of FAST TCP described by (8) and (9) in the following theorem.

Theorem 1: FAST TCP under congestion, described by (8) and (9), is globally exponentially stable; that is, $\tilde{z}_i(t)$ in (13) and $\tilde{p}_M(t)$ in (14) exponentially converge to zero from any initial condition.

Proof: Let

$$V(\tilde{z}(t)) = \frac{1}{2\gamma} \tilde{z}^T(t) A \tilde{z}(t) \tag{23}$$

be a Lyapunov function candidate, where A is a positive definite matrix defined in (17). Taking the derivative of (23) along the solutions of the system (16) and using the diagonal matrix D defined in (21), we obtain

$$\begin{aligned} \dot{V}(\tilde{z}(t)) &= \frac{1}{2\gamma} \left(\dot{\tilde{z}}^T(t) A \tilde{z}(t) + \tilde{z}^T(t) A \dot{\tilde{z}}(t) \right) \\ &= -\tilde{z}^T(t) A \tilde{z}(t) + \tilde{z}^T(t) \left(B(t) - C(t) \right) \tilde{z}(t - T_i(t)) \\ &\left(\because\ (16) \text{ and } \dot{\tilde{z}}^T(t) A \tilde{z}(t) = \tilde{z}^T(t) A \dot{\tilde{z}}(t) \right) \\ &\leq -\tilde{z}^T(t) A \tilde{z}(t) + \frac{1}{2} \tilde{z}^T(t) \left(B(t) - C(t) \right) \tilde{z}(t) \\ &\quad + \frac{1}{2} \tilde{z}^T(t - T_i(t)) \left(B(t) - C(t) \right) \tilde{z}(t - T_i(t)) \\ &(\because (B(t) - C(t)) \text{ is positive semidefinite by Lemma 3}) \\ &\leq -\tilde{z}^T(t) A \tilde{z}(t) + \frac{1}{2} \tilde{z}^T(t) B(t) \tilde{z}(t) \\ &\quad + \frac{1}{2} \tilde{z}^T(t - T_i(t)) B(t) \tilde{z}(t - T_i(t)) \\ &(\because\ C(t) \text{ and } (B(t) - C(t)) \text{ is positive semidefinite} \\ &\text{by Lemma 3}) \\ &< -\frac{1}{2} \tilde{z}^T(t) A \tilde{z}(t) - \frac{1}{2} \tilde{z}^T(t) D \tilde{z}(t) + \frac{1}{2} \tilde{z}^T(t) B(t) \tilde{z}(t) \\ &\quad + \frac{1}{2} \tilde{z}^T(t - T_i(t)) B(t) \tilde{z}(t - T_i(t)) \\ &(\because\ (A - D) \text{ is positive definite by Lemma 4}) \\ &< -\frac{1}{2} \tilde{z}^T(t) A \tilde{z}(t) + \frac{1}{2} \sup_{t-T^M \leq s \leq t} \tilde{z}^T(s) D \tilde{z}(s) \\ &(\because\ (D - B(t)) \text{ is positive definite by Lemma 4}) \\ &\leq -\frac{1}{2} \tilde{z}^T(t) A \tilde{z}(t) + \frac{1}{2} \delta \sup_{t-T^M \leq s \leq t} \tilde{z}^T(s) A \tilde{z}(s) \\ &(\because\ (22) \text{ in the proof of Lemma 4}) \\ &= -\gamma V(\tilde{z}(t)) + \gamma\delta \sup_{t-T^M \leq s \leq t} V(\tilde{z}(s)), \end{aligned} \tag{24}$$

where $\delta < 1$ by Lemma 4, and $\gamma > \gamma\delta$. Then, from Lemma 2, it follows that $V(\tilde{z}(t)) = \frac{1}{2\gamma} \tilde{z}^T(t) A \tilde{z}(t)$ exponentially converges to zero, which implies that $\tilde{z}_i(t)$ in (15) exponentially converges to zero for all $1 \leq i \leq N$. Moreover, since $T_i(t) < T^M$ for all $1 \leq i \leq N$, it follows that $\tilde{z}_i(t - T_i)$ exponentially converges to zero, which guarantees that $\tilde{p}_M(t)$ exponentially converges to zero in view of (14). □

5. Simulation

We simulate the FAST TCP model described by (3) and (5) with *MATLAB*, and exemplify the global exponential stability of FAST TCP. Furthermore, we simulate FAST TCP with

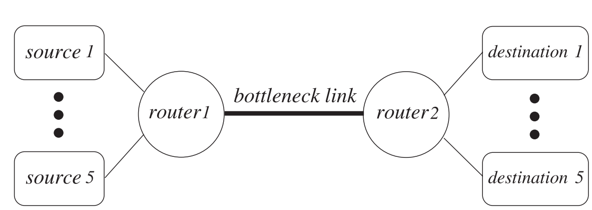
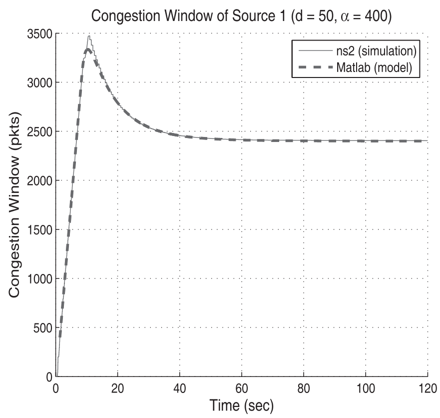


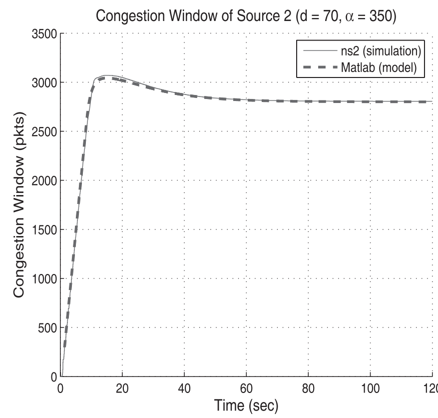
Fig. 2 Network topology.

Table 1 Equilibrium points.

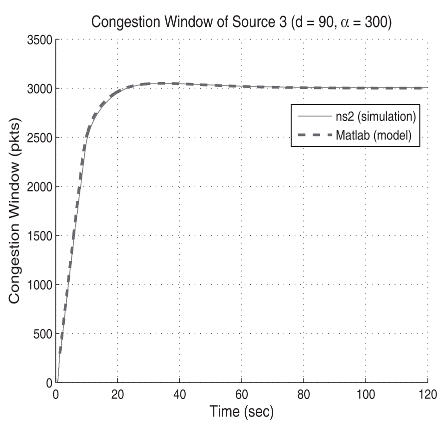
c (pkts/ms)	d_i (ms)	α_i (pkts)	Congestion window (z_i^*) (pkts)	Queuing delay (q_M^*) (ms)
	50	400	2400	
	70	350	2800	
	90	300	3000	10
150	130	250	3500	
	150	200	3200	



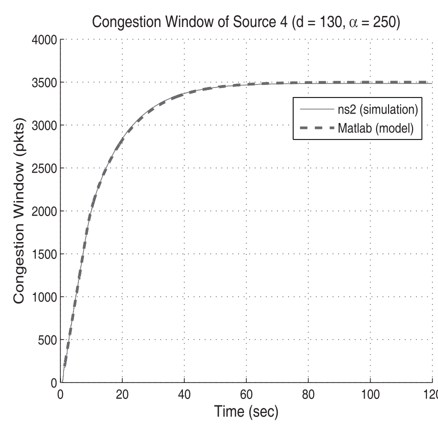
(a) Congestion window of FAST TCP source 1



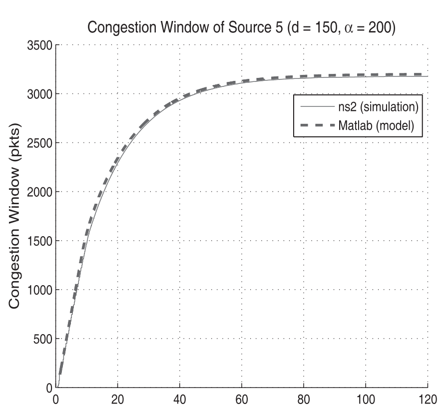
(b) Congestion window of FAST TCP source 2



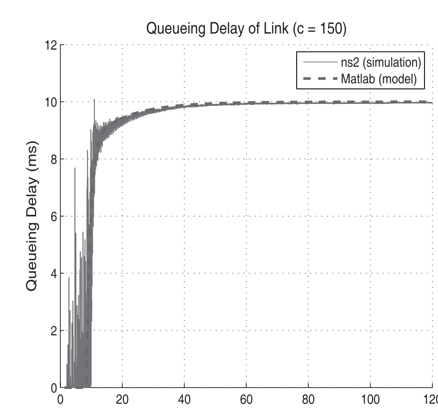
(c) Congestion window of FAST TCP source 3



(d) Congestion window of FAST TCP source 4



(e) Congestion window of FAST TCP source 5



(f) Queuing delay of link

Fig. 3 Congestion windows and queuing delay from *MATLAB* and *ns-2* simulation results.

ns-2 network simulator [20], and demonstrate the global exponential stability of the real FAST TCP implementation code that is found at [21]. Note that *ns-2* simulator does not simulate the FAST TCP model (3) and (5), but does simulate the real FAST TCP implementation code. Hence, the *ns-2* simulation results can be regarded as experiment results of FAST TCP.

We conduct the simulations for a network with a single bottleneck link shared by five heterogeneous sources as shown in Fig. 2. The link capacity and the constant round trip propagation delays are set as $c = 150$ pkts/ms and $d_1 = 50$ ms, $d_2 = 70$ ms, $d_3 = 90$ ms, $d_4 = 130$ ms, $d_5 = 150$ ms. The congestion control parameters are set as $\alpha_1 = 400$, $\alpha_2 = 350$, $\alpha_3 = 300$, $\alpha_4 = 250$, $\alpha_5 = 200$, which are fairly small values in comparison with the bandwidth-delay product of each source.

Table 1 summarizes the calculated equilibrium points of the link's queuing delay and the sources' congestion windows. Figure 3 shows the congestion windows and queuing delay of the simulation results, where the broken lines indicate the *MATLAB* results and the solid lines indicate the *ns-2* results. All figures illustrate that each source's congestion window is bounded and exponentially stable in view of both the model and the real implementation code of FAST TCP. Moreover, the similarity of the shapes between *MATLAB* and *ns-2* results in Fig. 3 illustrates that the proposed model (3) and (5) for the FAST TCP sources and the queuing delay adequately describe real FAST TCP networks.

6. Conclusion

In this paper, we address the global exponential stability of FAST TCP in a single-link multi-source network in the presence of time-varying network feedback delay. Based on a continuous-time model for the FAST TCP sources and the queuing delay at the link, we show that FAST TCP in itself is globally exponentially stable without any specific conditions on each source's control parameter α_i or the update gain γ. It is remarkable that FAST TCP is globally exponentially stable even in the presence of time-varying network feedback delay that is directly dependent on the queuing delay.

As a future work, it is worth while to create a dynamic model for other TCP algorithms and analyze the stability characteristics based on the dynamic model. The obtained stability characteristics can be used to tune the parameters or modify a part of algorithm for stable and efficient operations of each TCP algorithm.

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Appendix

Proof of Lemma 3: We note that $\alpha_i > 0$ and $d_{ip} := d_i + p_M(t) > 0$ for all $1 \leq i \leq N$, and obtain from (18) and (19) the following equation:

$$\left(\sum_{k=1}^{N} \frac{\alpha_k}{d_{kp}}\right) x^T (B(t) - C(t)) x = \sum_{k=1}^{N} \frac{\alpha_k}{d_{kp}} \sum_{k=1}^{N} \frac{x_k^2}{\alpha_k d_{kp}} - \left(\sum_{k=1}^{N} \frac{x_k}{d_{kp}}\right)^2 \qquad \text{(A· 1)}$$

Now, we will prove that (A· 1) is non-negative by using the mathematical induction with respect to N. First, for the case of $N = 1$, (A· 1) becomes zero as $\frac{\alpha_1}{d_{1p}}\frac{x_1^2}{\alpha_1 d_{1p}} - \left(\frac{x_1}{d_{1p}}\right)^2 = 0$, which shows that (A· 1) is non-negative when $N = 1$. Next, we suppose that (A· 1) is non-negative for $N = n$ and $1 < n < N$, that is, $\left(\sum_{k=1}^{n}\frac{\alpha_k}{d_{kp}}\sum_{k=1}^{n}\frac{x_k^2}{\alpha_k d_{kp}} - \left(\sum_{k=1}^{n}\frac{x_k}{d_{kp}}\right)^2\right) \geq 0$, and develop the following derivation for $N = n + 1$:

$$
\begin{aligned}
&\sum_{k=1}^{n+1}\frac{\alpha_k}{d_{kp}}\sum_{k=1}^{n+1}\frac{x_k^2}{\alpha_k d_{kp}} - \left(\sum_{k=1}^{n+1}\frac{x_k}{d_{kp}}\right)^2 \\
&= \left(\sum_{k=1}^{n}\frac{\alpha_k}{d_{kp}} + \frac{\alpha_{n+1}}{d_{(n+1)p}}\right)\left(\sum_{k=1}^{n}\frac{x_k^2}{\alpha_k d_{kp}} + \frac{x_{n+1}^2}{\alpha_{n+1}d_{(n+1)p}}\right) \\
&- \left(\sum_{k=1}^{n}\frac{x_k}{d_{kp}} + \frac{x_{n+1}}{d_{(n+1)p}}\right)^2 \\
&= \left(\sum_{k=1}^{n}\frac{\alpha_k}{d_{kp}}\sum_{k=1}^{n}\frac{x_k^2}{\alpha_k d_{kp}} - \left(\sum_{k=1}^{n}\frac{x_k}{d_{kp}}\right)^2\right) \\
&+ \frac{1}{d_{(n+1)p}}\sum_{k=1}^{n}\left[\frac{1}{d_{kp}}\left(\frac{\alpha_k}{\alpha_{n+1}}x_{n+1}^2 - 2x_{n+1}x_k + \frac{\alpha_{n+1}}{\alpha_k}x_k^2\right)\right] \\
&= \left(\sum_{k=1}^{n}\frac{\alpha_k}{d_{kp}}\sum_{k=1}^{n}\frac{x_k^2}{\alpha_k d_{kp}} - \left(\sum_{k=1}^{n}\frac{x_k}{d_{kp}}\right)^2\right) \\
&+ \frac{1}{d_{(n+1)p}}\sum_{k=1}^{n}\left[\frac{1}{d_{kp}}\left(\sqrt{\frac{\alpha_k}{\alpha_{n+1}}}x_{n+1} - \sqrt{\frac{\alpha_{n+1}}{\alpha_k}}x_k\right)^2\right] \geq 0,
\end{aligned}
$$

which completes the proof for the inequality $x^T(B(t) - C(t))x \geq 0$. □

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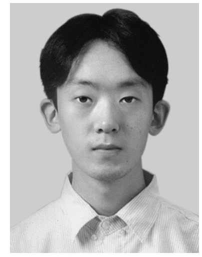

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