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Markov model of link connectivity in mobile ad hoc networks

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Abstract This paper proposes a Markov model of link connectivity for mobile ad hoc networks. Under a random behavior, the model provides a unified approach to describe many different mobility models including entity mobility models and group mobility models. Using the model, we can predict the time dependence of link connectivity, and estimate a settling time for which node movements are considered in a transient state. We verify the model with the simulation results of four different mobility models using a global connectivity and a link duration distribution.

Keywords Mobile ad hoc network · Link duration · Continuous space stochastic process

1 Introduction

During the past decade, advances in wireless communication have led the promises of ubiquitous computing and flexible user mobility. To support the promises, a new network paradigm was devised, in which network structures are virtually constructed on an as-it-needed basis, referred to as *ad hoc networks*. A mobile node in the networks does not require any fixed infrastructure for communication as needed in cellular communication networks. Instead, a mobile node can cooperate with other mobile nodes, and becomes an active switch to forward network traffic to other nodes. The node can dynamically adjust its role in the networks depending on its link status and movement.

For designing a routing algorithm of the networks, a movement pattern has a great impact on many aspects of the network performance such as holding time, capacity planning, location services, handover management, and path discovery. For example, a node special distribution as a result of the node movement pattern is the key information to find an optimal transmit power to control connectivity properties of the network [6]. Modeling and synthesizing precise and realistic user-mobility in the highly dynamic network plays a very important role on designing an efficient methodology and optimizing its investment, because performance measurement based on an incorrect model will result in invalid conclusion.

A link between two nodes in ad hoc network becomes active if they receive signal power above a system-dependent threshold. The received signal power has a close relation with the distance from a source node as well as obstacles, a fading effect, and many other factors. When they move away out of their transmission range, the link becomes deactivated before they come close within a transmission range again.

Consequently, the node movement produces a sequence of on-off link changes. This paper develops a generalized framework to analyze the link connectivity modeling the on-off transition as a Markov state machine. The framework characterizes different mobility models by describing the link connectivity as a function of time regardless the detailed movement pattern. The paper shows simulation results with four mobility models to verify the model that demonstrates the initial unstability of some mobility models.

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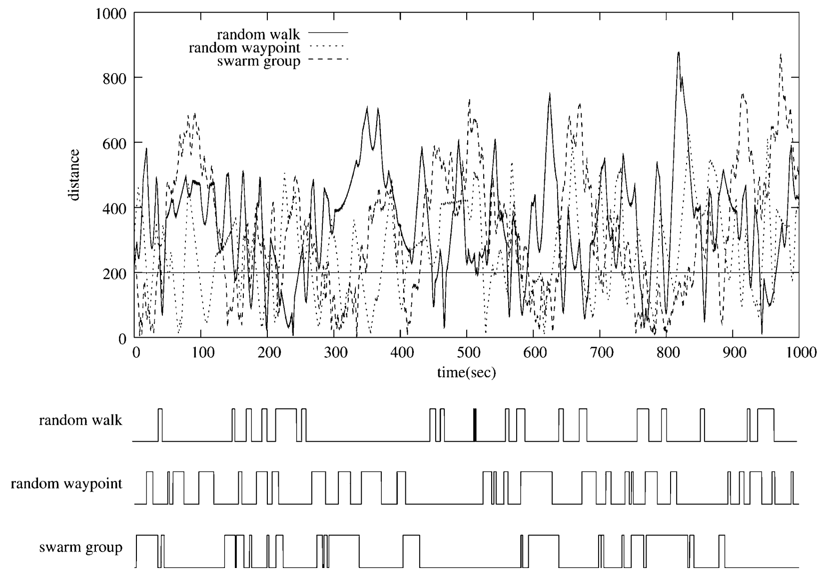

Fig. 1 Time-varying inter-nodal distance with transmission range 200 m

2 Markov model of link connectivity

Among many mobility metrics, a link expiration time and a link connectivity are the widely adopted metrics in mobile wireless network community [11, 12]. The link expiration time is a period of time that two nodes can communicate with each other directly without losing the connection at any instance. This quantity can be predicted in a very limited circumstance such as a constant nodal speed and a random behavior in the change of direction during the time observed [8, 15].

The upper part of Fig. 1 shows time-depending inter-node distance between two arbitrary nodes from three mobility models. In mobile ad hoc networks, pairs of nodes have a direct connection when they are mutually within their transmission ranges. Assuming the transmission range is 200 m, their connectivities are illustrated as on-off graphs in the lower part of the figure, where the higher values indicate the nodes in the range so that they can communicate directly, but the low values represent the period when two nodes are out of the transmission range, or the loss of direct connection.

The inter-nodal distances in the random walk model has no pattern and seems to show a random behavior. McDonald and Znati [8] provided a mathematical model to discover the distribution of aggregate distance and direction covered by a node in the random walk model. Using the randomness of speed and direction of a node, they formulated the probability that a node stays within a 2-D circular region and approximated the aggregate distance as a Raleigh distribution. They assumed that nodes are placed on 2-D plain surface, in which they are distributed uniformly in the space, and that they have perfect circular transmission range moving independently from other nodes.

For the random waypoint model, the inter-nodal distance seems to be arbitrary, but the aggregate connection time is longer than the one of random walk model. The assumptions used in the previous analysis cannot be applied to this mobility model, because the spatial node distribution in a random waypoint model is non-uniform. The node density at the center of simulation area is much higher than the one at the boundary [2, 9, 14]. For the swarm group mobility model where the nodal movement is affected by its neighbors, the independent movement assumption cannot be held [7, 13]. For a RPGM mobility model, each group has a logical center and nodal movements in the same group are determined by the motion of its logical center [5].

Moreover, it is also unclear how a mathematical analysis can be applied in urban area, where a geographical terrain and the obstacles of wireless channels make non-circular transmission range. In a military application of ad hoc network, a platoon of soldiers move in a cooperative manner based on their field manuals, but not independently. To overcome these problems, we need a general framework to be able to analyze various mobility models, which can characterize real movement patterns.

2.1 Stationary distribution of link connectivity

To the goal of the simplicity, we assume that the connectivity of two nodes is a memoryless stochastic process, meaning that a future connectivity of two nodes is independent of its history, but depending only on the current state of the connectivity. In other words, the connectivity at time $t + \Delta t$ is depending on the state of the link at time t, but independent of the state during $[0, t)$. In Fig. 2, we describe the connectivity between two nodes in ad-hoc network and its transition probability. The Markovian chain contains a state space $S = \{D, C\}$, where the state D indicates that two

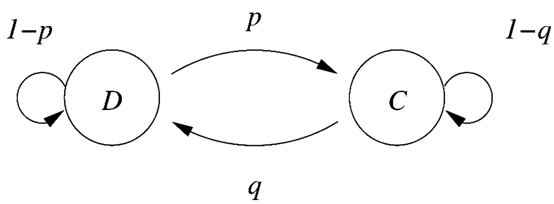

Fig. 2 Connectivity states and their transition probability

nodes are disconnected, but in the state C, they are connected with a bidirectional wireless channel. The connection state at the next time slot has the probability p for the transition from D to C, but the probability q for the transition from C to D. Therefore, the state transition probability matrix P is given as

$$P = \begin{bmatrix} 1-p & q \\ p & 1-q \end{bmatrix} \quad (1)$$

This is the general form for a transition matrix of a two state Markovian chain. To describe the long-term behavior of the connection, the behavior of P_n for a large n value needs to be calculated. The matrix P has Eigenvalues 1 and $1-p-q$ if $0 < p, q < 1$. Because of $|1-p-q| < 1$, the stationary probability of the transition matrix is given as

$$\lim_{n\to\infty} P^n = \begin{bmatrix} q/(p+q) & q/(p+q) \\ p/(p+q) & p/(p+q) \end{bmatrix} \quad (2)$$

From the equation, we know that the link remains in the connection state with $p/(p+q)$ ratio and in the disconnection state with $q/(p+q)$ ratio.

2.2 Continuous-time analysis of link connectivity

We have a 2-state space $S = \{D, C\}$, and define a process X_t that has the Markov property such as

$$P\{X_t = c|X_r, 0 \le r \le s\} = P\{X_t = C|X_S\},$$

and the process is time-homogeneous so that we obtain

$$P\{X_t = C|X_S = D\} = P\{X_{t-S} = C|X_0 = D\}$$

Considering λ and μ as transition rates of $D \to C$ and $C \to D$, respectively, a continuous-time Markov chain with λ and μ is a stochastic processes X_t satisfying

$$P\{X_{t+\Delta t} = C|X_t = C\} = 1 - \mu\Delta t$$
$$P\{X_{t+\Delta t} = C|X_t = D\} = \lambda\Delta t$$

Let $p_S(t)$ denote $P\{X_t = s\}$ for $s \in \{C, D\}$. Then, we can obtain

$$\begin{aligned} p_C(t+\Delta t) &= p_C(t)P\{X_{t+\Delta t} = C|X_t = C\} \\ &\quad + p_D(t)P\{X_{t+\Delta t} = C|X_t = D\} \\ &= p_C(t)(1-\mu\Delta t) + p_D(t)\lambda\Delta t p'_C(t) \\ &= \lambda p_D(t) - \mu p_C(t). \end{aligned}$$

Likewise,

$$p'_D(t) = -\lambda p_D(t) + \mu p_C(t).$$

Using a probability vector $\mathbf{p}(t)$ as $[p_D(t), p_C(t)]^T$, the above system can be simplified as below:

$$\mathbf{p}'(t) = A\mathbf{p}(t), \text{ and } A = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix}, \quad (3)$$

where the transition matrix A is called the infinitesimal generator of the chain. The differential equation has a solution as follow:

$$\mathbf{p}(t) = e^{At}\mathbf{p}(0), \quad \text{and}$$
$$e^{At} = L^{-1}[(sI - A)^{-1}]$$
$$= \begin{bmatrix} \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} & \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t} \\ \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} & \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t} \end{bmatrix}, \quad (4)$$

where L is the Laplace transform. If t goes to infinity, the matrix e^{At} is converged to a constant as

$$\lim_{t\to\infty} e^{At} = \begin{bmatrix} \frac{\mu}{\lambda+\mu} & \frac{\mu}{\lambda+\mu} \\ \frac{\lambda}{\lambda+\mu} & \frac{\lambda}{\lambda+\mu} \end{bmatrix},$$

hence we obtain

$$\lim_{t\to\infty} p_D(t) = \frac{\mu}{\lambda+\mu}, \quad \text{and} \quad \lim_{t\to\infty} p_C(t) = \frac{\lambda}{\lambda+\mu}$$

There are several requirements for describing a mobility model mathematically. One of the most important requirements is that the model should be *stationary* to derive the distribution of link and path duration for those mobility models [4]. An ad-hoc network simulation is expected to run for a long time, and to reach a steady state. Besides of the random walk model, however, it has not been known how long a mobility model take to reach a steady state. In the following section, we will describe a method to identify a transient time-period during which node movement exhibits temporal behavior.

2.3 Analysis of settling time

A modern control theory uses a rising time, a peak time, overshoot and settling time to describe the transient response of a system [3]. The settling time is one of the most important measures to characterize a time response of the system, which is defined as a time period required for the system output to settle within a certain percentage of a final value. In general, the system response is considered to be settled if the response remains within 2 to 5% of a final value.

The dynamics of the link-connection probability do not have a damping ratio, because Eq. (3) is a first-order derivative equation, and the output asymptotically converges to a stationary value. In this case, a settling time is a useful metric to describe the dynamic attribute of link-connection probability. As indicated by Eq. (4), the connection probability approaches to $\frac{\lambda}{\lambda+\mu}$ at the rate of $(\lambda p_D(0) - \mu p_C(0))e^{-(\lambda+\mu)t}$ from an initial probability $p_C(0)$. For a random walk, the stationary probability $\frac{\lambda}{\lambda+\mu}$ is supposed to be close to the initial connection probability so that the entire movement can be considered as a valid mobility model.

In other mobility models that reveal non-steady state such as a random waypoint model, the connection probability is in transient state for non-trivial period of time before it falls within some range of the stationary probability. The settling time (t_s) is mathematically derived as

$$\lim_{t\to\infty} |1 - p_C(t_s)/p_C(t)| \leq \varepsilon$$

By substituting Eq. (4) and $k = p_D(0) - p_C(0)\mu/\lambda$, we can obtain

$$|ke^{-(\lambda+\mu)t}| \leq \varepsilon, \quad \text{or} \quad t_s \geq \frac{1}{\lambda+\mu} \ln \frac{|k|}{\varepsilon} \tag{5}$$

3 Experiments

We present the quantative results of stability using four different mobility models, two models from entity mobility models (random walk and random waypoint) and two models from group mobility models (swarm group and RPGM). In the simulations, we assumed a small campus wireless network where people are walking around. A node velocity in this circumstance is relatively small compared with its simulation region and transmission range. It is possible to consider other situations that generate different node connectivity. For example, the node connectivity of vehicles traveling on highway could be significantly different from the campus network. A node velocity is much higher than in a campus network and a larger transmission range will be required. As a result, a larger simulation area is also desired. Otherwise, some probabilistic distortion can affect to the measurement of mobility model. Supposed that n nodes are randomly distributed in $R \times R$ simulation area and each node has the circular transmission range of r. Only $(1 - 2r/R)^2 n$ nodes maintain a full circular transmission in average, but other nodes have incomplete circular transmission due to the boundary. For example, a transmission range of 250 m in 1000 m by 1000 m area results in that only 25% of nodes maintains complete circle of transmission range. This distortion can seriously affect the measures of mobility models as shown in [1].

To compare different mobility models in a fair manner, the simulations use common simulation specifications as much as possible. Each simulation runs for 2 h 1000 × 1000 m area with one hundred nodes. Each node is initially placed at a random location except the RPGM model, where nodes are uniformly distributed within the geographic scope of a group. When a node hits a boundary, it is rebounded in three mobility models. Note that the rebounding does not occur in the random waypoint model. A node velocity is uniformly chosen in the range [1, 10] m/s and a traveling time is selected randomly from the range of [0, 10] s. For random waypoint model, a pause time is not considered and a destination is chosen randomly in the simulation area. A node movement in the swarm group model was determined under consideration of the target position and neighbor's positions. Each node in the model tends to follow the nearest target movement and to avoid collision with its neighbor nodes, and node velocity is a weighted sum of these tendencies [7]. The RPGM model considers two groups, each group contains 50 nodes [5]. One hundred destination points are randomly selected and a traveling time to each destination point was set to 72 s to make the total simulation time for 2 h.

The stationary connection ratio $p/(p+q)$, or $\lambda/(\lambda+\mu)$, can be computed by counting the ratio of the link connections. From the Markov model, the arrival rate μ is obtained from a mean link connection time.

Figure 3 shows the link connection probability of the random walk model. Raw data has a wide variance but its time average is converged quickly. The probability variation of analytic result is less than 0.0003, which is less than 2% of the final output value. It concludes that the settling time of the random walk model is negligible.

$$F(t) = P(T_C \leq t) = 1 - P(T_C > t) = 1 - e^{-\mu t} \quad \text{for } t > 0$$
$$E(T_C) = \int_0^\infty t dF(t),$$

where T_C is a connection time, $F(\cdot)$ is a survival function, and $E(\cdot)$ is the mean of connection time. The settling time of each model is obtained using Eqs. (4) and (5). The initial connectivity $p_C(0)$ in Eq. (4) is computed using the relation between the simulation area and the transmission area.

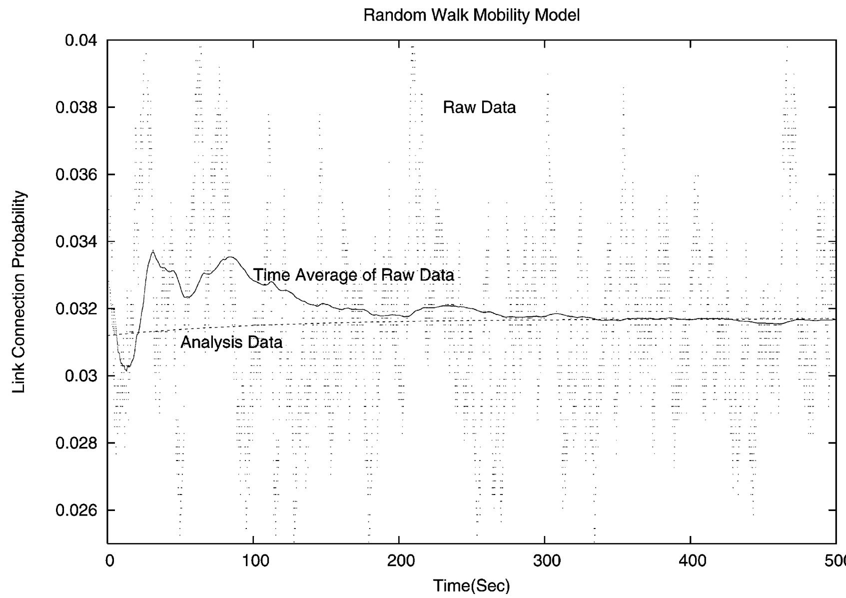

Fig. 3 The link connection probability for random walk mobility models with transmission range 100 m

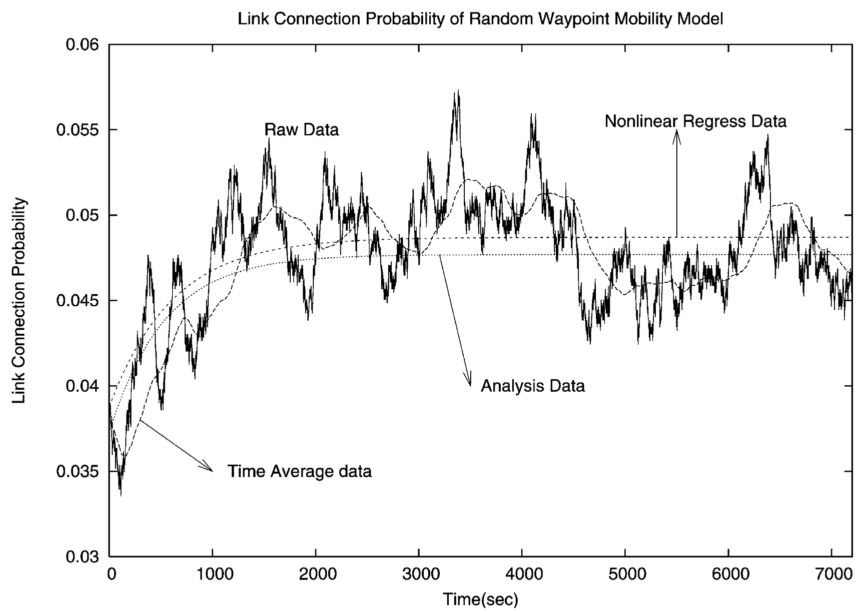

Fig. 4 The link connection probability for random waypoint mobility models with transmission range 100 m

Figure 4 shows the connection probability of the random waypoint model. As the simulation progresses, the probability increases. The reason of improving connectivity is the non-uniform nodal distribution of the model, and the nodal density in the center region is higher than one at the boundary. In addition, nodes congregate more in the center as the time goes on [14]. Consequently, the model needs non-trivial settling time before showing its stability. The nonlinear regress data in the figure is derived by assuming that the nonlinear function of the raw data follows Eq. (4) using a least square error method.

In Fig. 5, the connection probability of the swarm group model is illustrated. The probability is increasing while the simulation goes on similar to the random waypoint model, but not because of the non-uniform nodal distribution. The nodal density of the model is very close to a uniform distribution [7]. In the simulation each node is initially placed at a random location. Shortly after the simulation starts, nodes tend to come together because of the natures of target-seeking, centroid, and velocity-matching. Until the binding forces balance dispersing forces such as collision-avoidance and random forces, the model needs to take a significant time.

Figure 6 shows the link connection probability of the RPGM model. The connectivity sharply increases intermittently caused by approaching two group leaders. As described previously, each group leader travels for a given time period toward a destination. In the simulation, the first cross of two groups occurs at 120 s, and the second cross at 180 s. We can observe that the cross movements produce the transient improvement of the connectivity. Besides of the

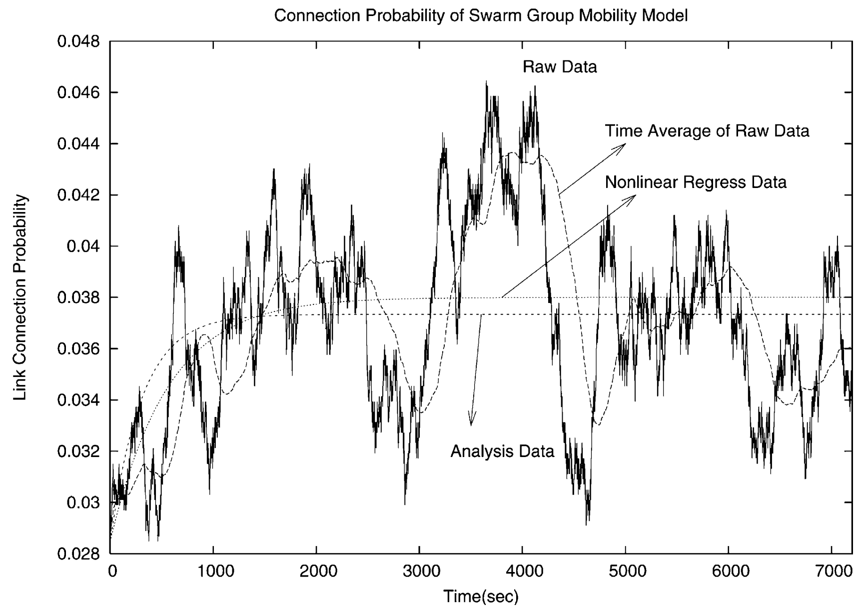

Fig. 5 The link connection probability for swarm group mobility models with transmission range 100 m

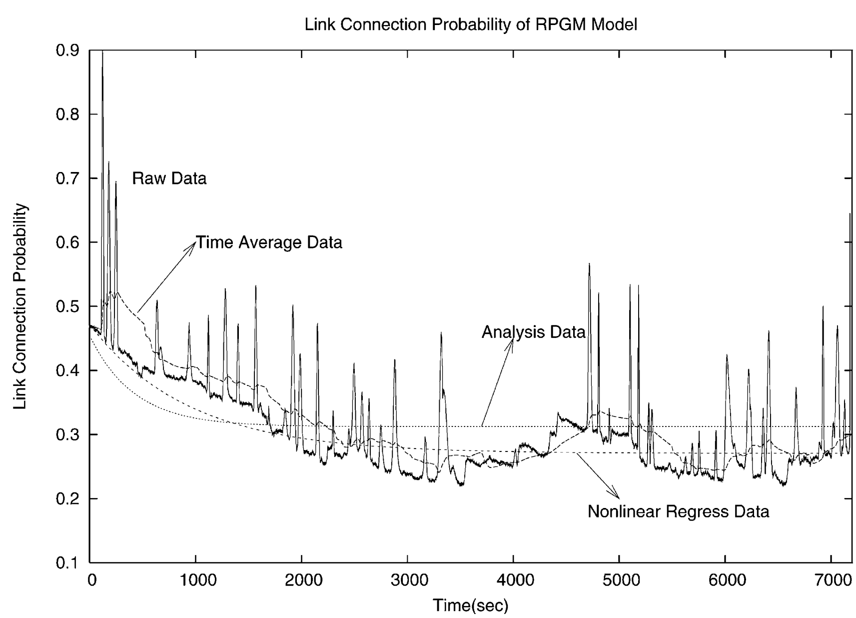

Fig. 6 The link connection probability for RPGM model with transmission range 100 m

temporal behavior, the RPGM shows a gradual decrement of the connectivity. The node density in each group is becoming sparse as the simulation goes on, because of the randomness of the model. Non-leader nodes move away from a group center as a result of random vector accumulation, and the model may require a significant time before reaching a steady state.

Table 1 summarizes the simulations and the analysis of the results. The random walk model shows the zero settling time, which is clearly understood because the model is supposed to present a random behavior theoretically regardless of the time the node is observed. In the meanwhile, the random waypoint, swarm group, and RPGM models require considerable time to reach to the steady state, even though nodes in these models are placed randomly at the beginning, but their locations are determined by some non-random characteristics. In the random waypoint mobility model, the spatial distribution is known to be distributed unevenly [10] and the average node velocity is also varying on time [14]. As a while, these non-uniform properties of the model make the link connectivity increase on time. Even though the spacial distribution of swarm group model is known to be near-uniform, member nodes moves in the region as a collaboration among their neighbors, resulting in dependent node movement [7]. In RPGM model, all member nodes are distributed near their logical center position at the beginning of simulation. Consequently, node connectivity of RPGM model is much higher than other models. Each node, however, keeps moving away from its group center because the movement of group member nodes is determined not only by the group center but also an arbitrary random velocity vector.

Table 1 The statistic values with transmission range 100 m

	Random walk	Random waypoint	Swarm group	RPGM
Connect ratio: $\lambda/(\lambda+\mu)$	0.03171	0.04837	0.03805	0.3126
Disconnect ratio: $\mu/(\lambda+\mu)$	0.96829	0.95163	0.96195	0.6874
μ	0.00873	0.00269	0.00311	0.00164
λ	0.00029	0.00013	0.00013	0.00074
t_S (sec): $\varepsilon = 2\%$	0.0	932.48	758.91	1304.43
t_S (sec): $\varepsilon = 5\%$	0.0	792.18	477.69	921.04

4 Conclusion

There have been several efforts to analyze the node movement and link connectivity based on geometric methods. Because the geometric method can be applied only to a specific condition, it has a difficulty in applying the method to other mobility models. This paper analyzed a link connectivity in mobile ad hoc networks based on a Markov model. The proposed method abstracted the link connectivity as on-off state transition without regarding the detail of node movement model. By deriving the link connectivity as a function of time, the method provided a general framework to estimate a settling time for a non-steady state mobility model such as random waypoint and group models. We also presented simulation results to verify our method. In the simulation, the random walk exhibited the zero settling time as expected. On the other hand, other mobility models presented the significant settling times.

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